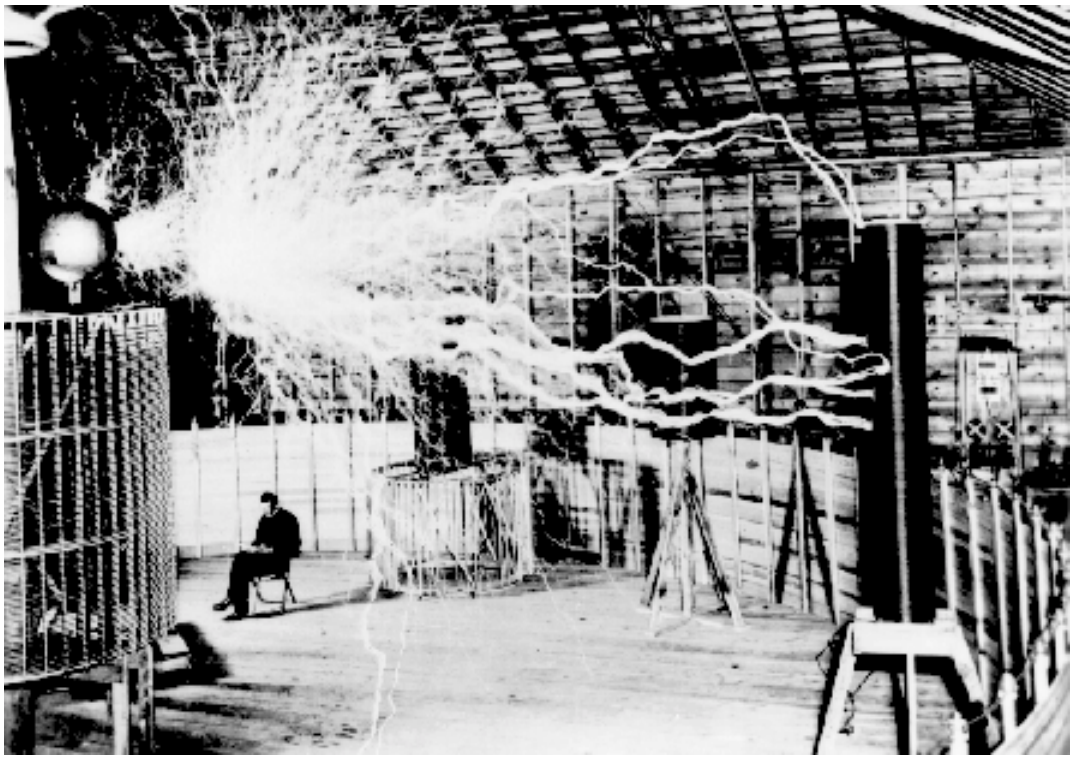
Chapter 16

Electric Charges, Electric Forces, and the Electric Field



There are two kinds of electric charge

- positive
- negative

Like charges repel and unlike charges attract.

Electric charge is conserved

- In any process the total amount of charge remains the same.
- Charge can move from one place to another, but the total amount of charge always remains the same.
- Charge can be created and destroyed so long as the net amount of charge does not change.

Electric charge is quantized

- It comes in discrete amounts.
- The smallest "amount" of charge is that contained in one electron or one proton. This is an amount of $e = 1.60 \times 10^{-19}$ C where "C" stands for Coloumb, the SI unit of electric charge. Charge is usually measured in millicoulombs (mC) 1×10^{-3} or microcoloumbs (μC) 1×10^{-6}.
- When an atom loses or gains an electron, it gains a net charge and is called an *ion*.

Conductors easily conduct electricity (allow electrons to flow easily), and insulators do not easily conduct electricity.

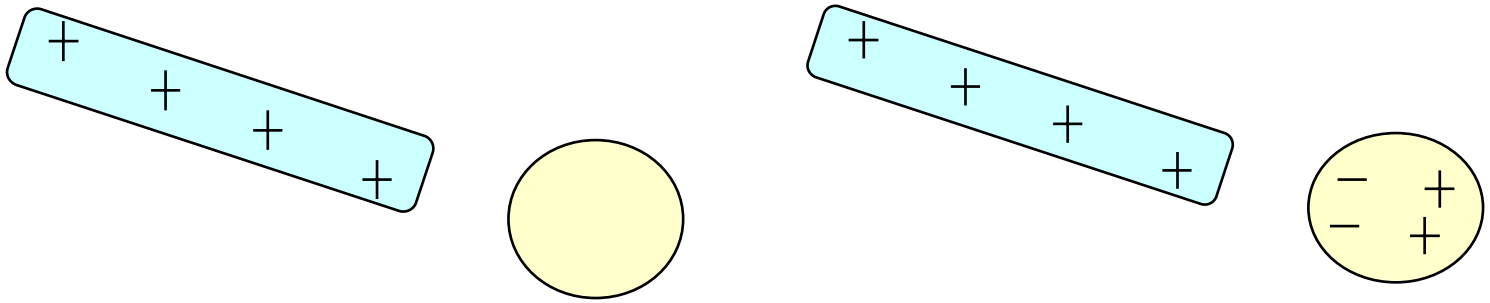
Interactive Question

Object A has a net positive charge, object B a net negative charge, and object C is neutral. Which of the following statements is true?

A) Object B has more electrons than object A
B) Objects A and B must be conductors
C) Object A has more protons than object C
D) Object A consists of only protons and no electrons
E) None of the above

Interactive Question

A charged object is brought close to an uncharged object. The uncharged object is attracted to the charged object due to *induced polarization*.



The uncharged object is:

A) a conductor
B) an insulator
C) either a conductor or an insulator

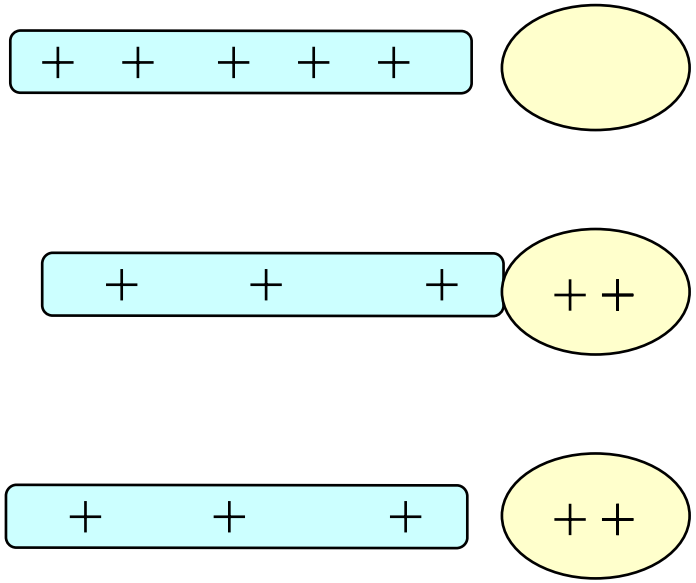
Interactive Question

Which Statement *best* describes why tiny bits of paper are attracted to a charged rubber rod?

A) Paper is naturally a positive material
B) Paper is naturally a negative material.
C) The paper becomes polarized by induction.
D) Rubber and paper always attract each other.
E) The paper acquires a net positive charge by induction.

There are at least two ways for a neutral object to become charged.

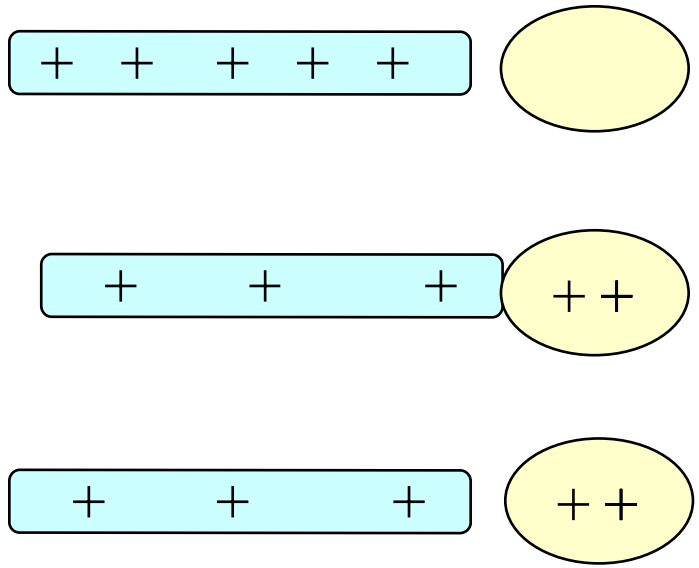
1) By conduction



If the two objects are identical in composition, size, and shape, then after touching each other they will both have exactly the same amount of charge on them.

Interactive Question

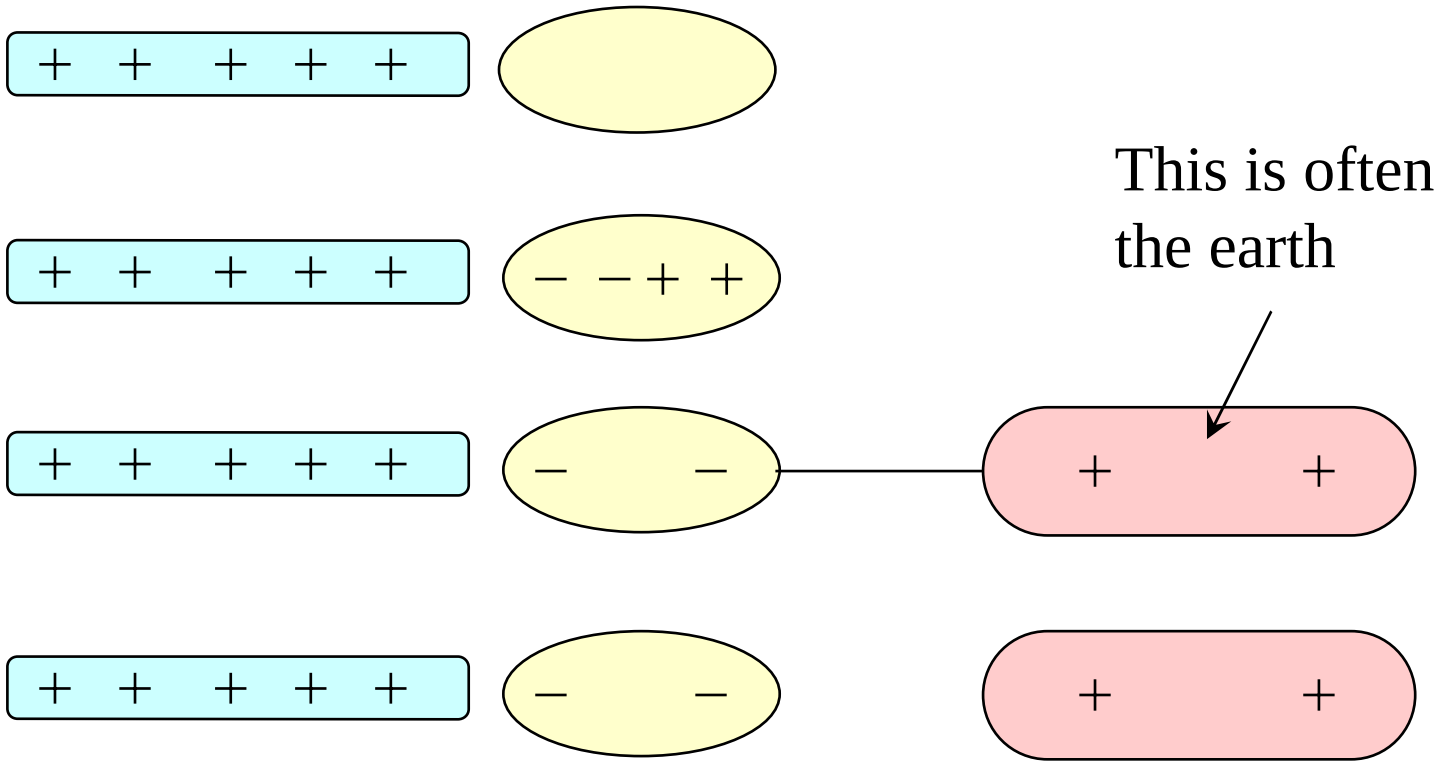
An initially neutral object is charged by conduction. This object is:



A) a conductor
B) an insulator
C) either a conductor or an insulator

There are at least two ways for a neutral object to become charged.

1) By conduction
2) By induction



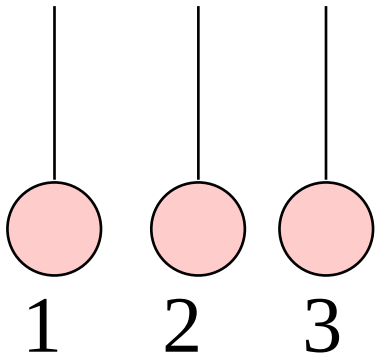
Interactive Question

Three styrofoam balls are suspended from insulating threads. Several experiments are performed and the following observations made:

I) Ball 2 attracts 1, but has no effect on ball 3.
II) Ball 1 is attracted to a negatively charged rod.

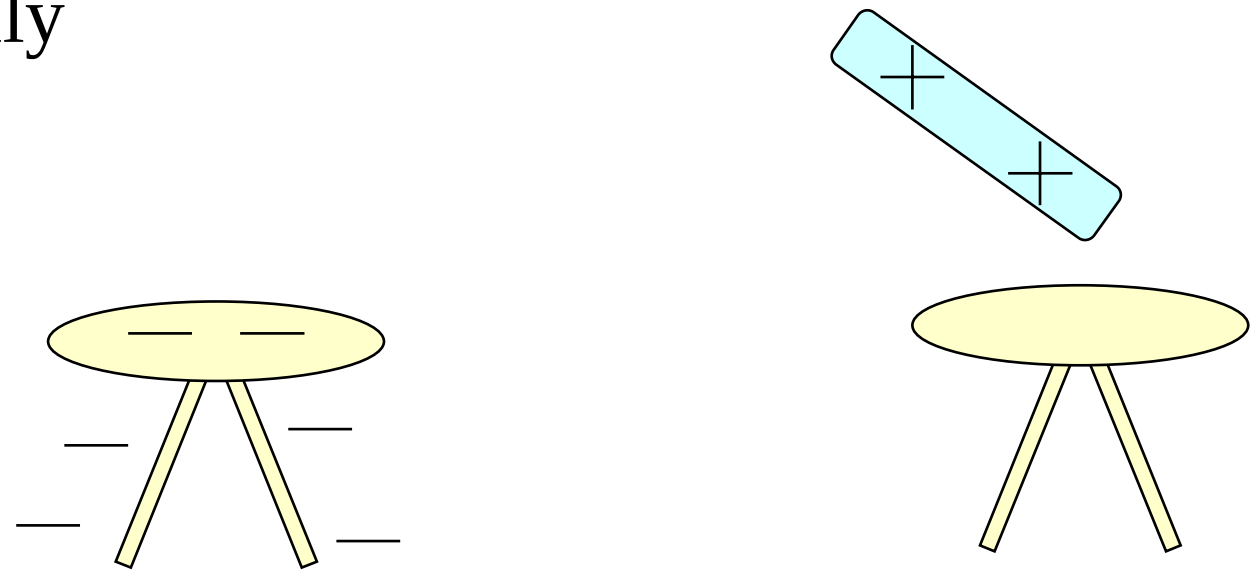
What are the charges on the balls?

	1	2	3
A)	+	–	0
B)	0	0	0
C)	+	0	0
D)	+	+	–
E)	0	+	–



Interactive Question

An electroscope is initially given a negative charge. When a rod that is positively charged is brought near to the electroscope, but does not touch it, the leaves will eventually

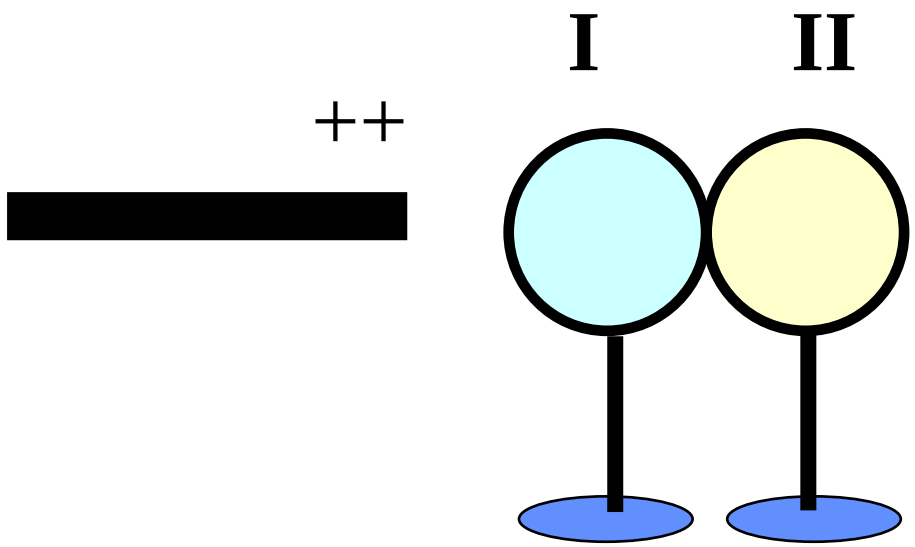


A) move closer together.
B) move farther apart.
C) remain where they are.

Interactive Question

Two uncharged spheres, **I** and **II**, are at rest on insulating stands. A positively charged rod is held *near to, but not touching* sphere **I**. While the rod is in place, the two spheres are separated.
How will the spheres be charged, *if at all?*

	I	**II**
A)	+	+
B)	+	–
C)	–	+
D)	0	+
E)	0	0



How would you determine

1. If an object is charged?
2. If an object is neutral?
3. If there is or is not a 3rd kind of charge?

Coulomb's Law

Coulomb's law describes the electric force between two "point" objects that are at rest.

$$\mathbf{F}_{21} = -\mathbf{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} \qquad \text{or} \qquad F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1||q_2|}{r^2}$$

q_1 is the charge of one object

q_2 is the charge of one object

r_{12} is the distance between the two objects

$k = 8.988 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$

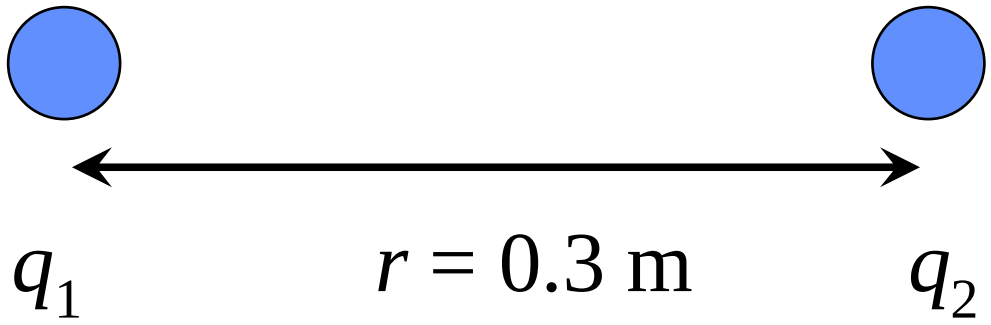
$k = 1/(4\pi\varepsilon_0)$ where ε_0 is the permittivity of free space

$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$

$$\hat{\mathbf{r}} = \frac{\mathbf{r}}{|\mathbf{r}|}$$

$\mathbf{r}_{12}$: Vector pointing from 1 to 2

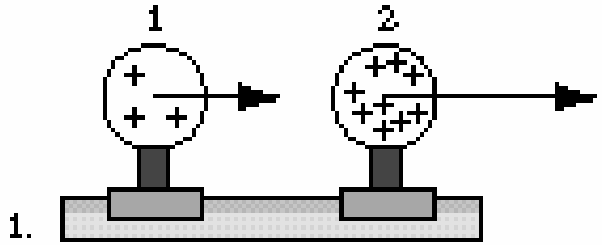
Problem: Two identical spheres hold different charges, $q_1 = 12 \times 10^{-9}$ C and $q_2 = -18 \times 10^{-9}$ C, and are separated by 0.3 m.(a) What is the electrostatic force on q_1 from q_2 and what is the force on q_2 from q_1? (b) q_1 and q_2 are then connected by a wire. Now what is the electrostatic force on q_1 from q_2 and what is the force on q_2 from q_1?



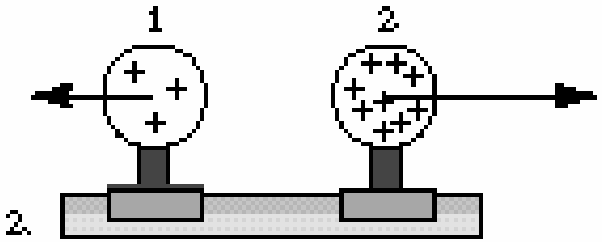
Interactive Question

Two uniformly charged spheres are fastened to and electrically insulated from frictionless pucks on an air table. The charge on sphere 2 is three times the charge on sphere 1. Which force diagram correctly shows the magnitude and direction of the electrostatic forces?

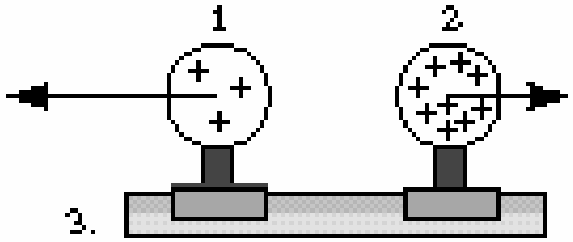
A)



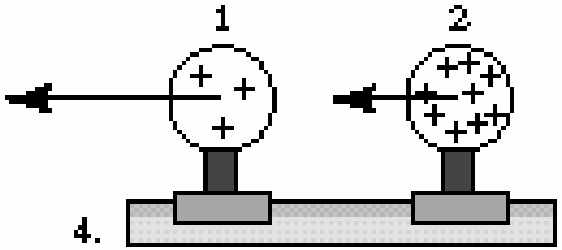
B)



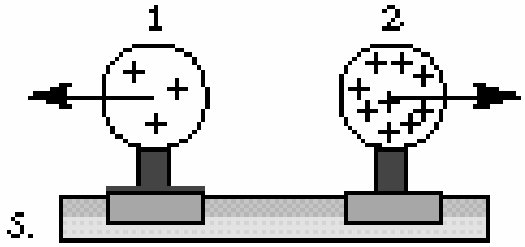
C)



D)

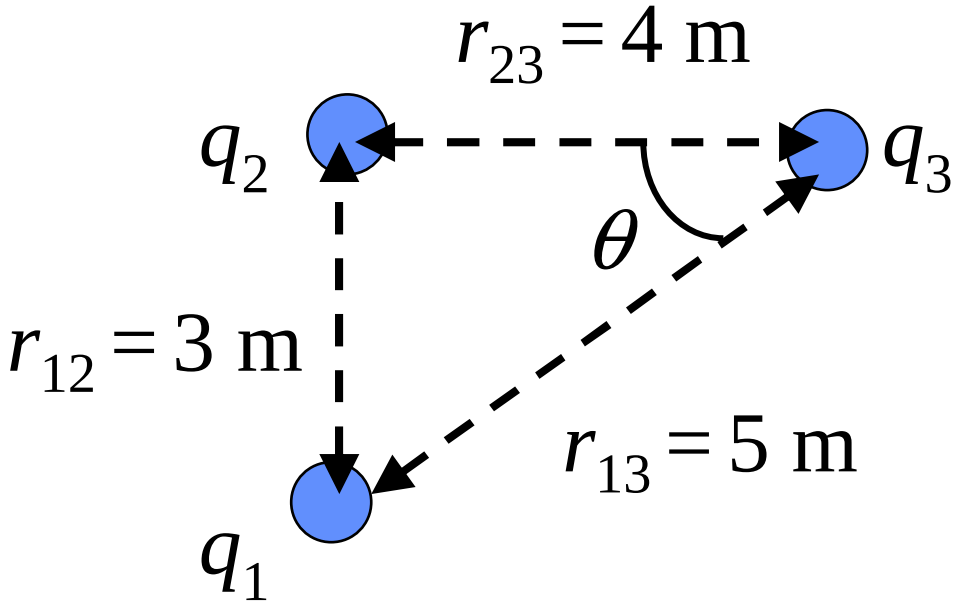


E)



The electric force is a vector, like all forces.

Problem: Three charged objects are arranged as shown, forming a right triangle. What is the total electrostatic force on the object labeled q_3?

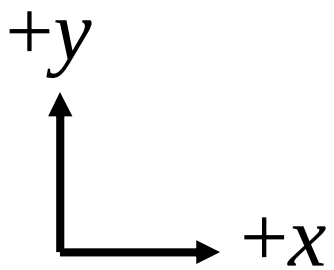


$q_1 = 6.00 \times 10^{-9}$ C

$q_2 = -2.00 \times 10^{-9}$

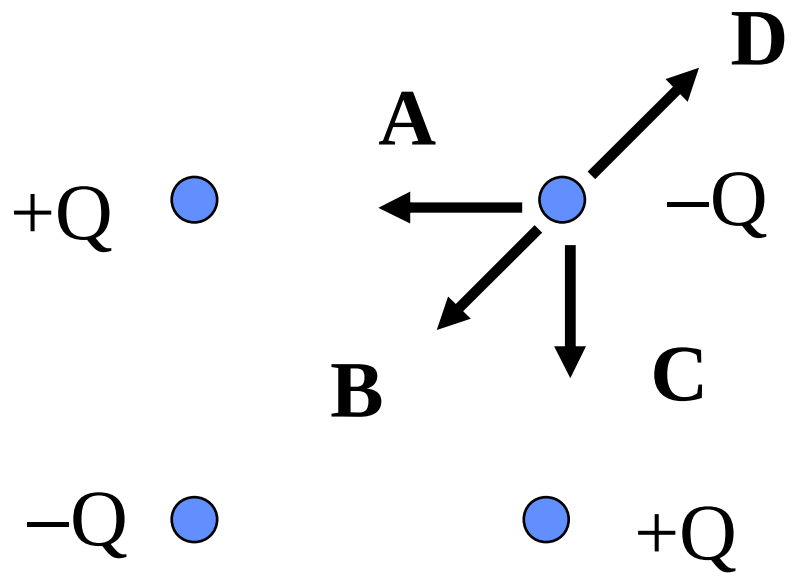
$q_3 = 5.00 \times 10^{-9}$ C

$\theta = \tan^{-1} 3/4 = 36.9°$



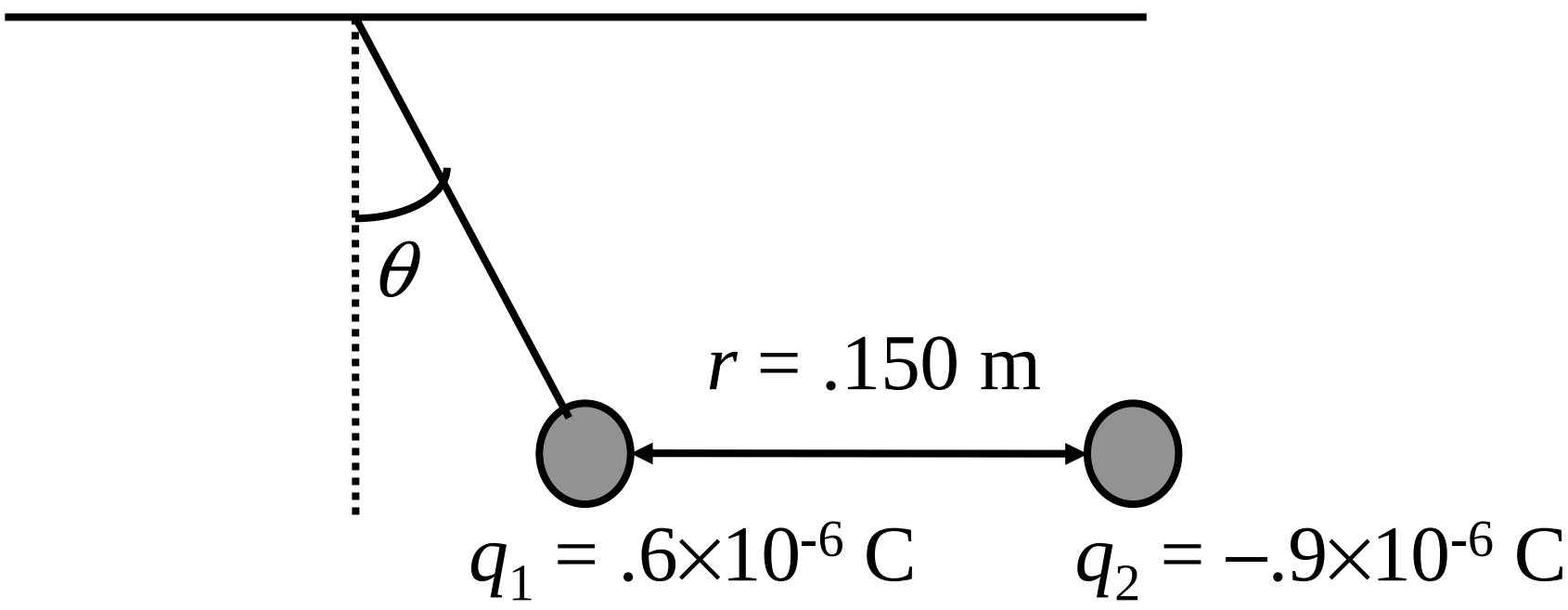
Interactive Question

Four point charges, each with the same magnitude, but with varying signs are arranged at the corners of a square. Which of the arrows shows the directions of the net force that acts on the charge in the upper right hand corner?



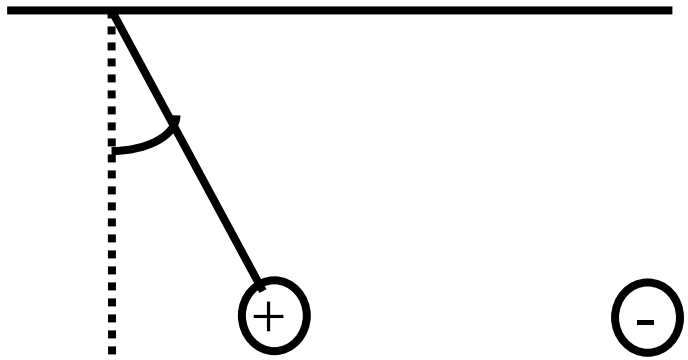
A) **A**
B) **B**
C) **C**
D) **D**
E) The net force on that charge is zero.

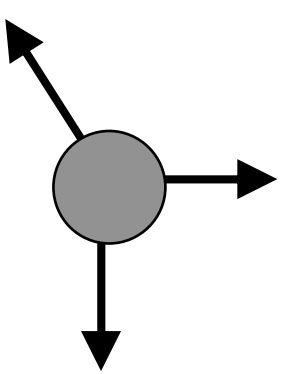
Problem: A charged object hangs from a string and is attracted to another charged object. What is the angle and tension in the string if the object hanging from it has a mass of $m = 8.0\times10^{-2}$ kg?



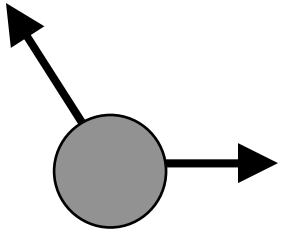
Interactive Question

The first step in solving any problem involving Newton's second law is to draw a free-body diagram. What is the correct free body diagram for the ball attached to the string in this problem?

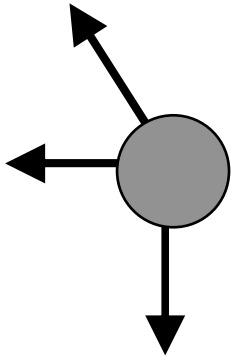




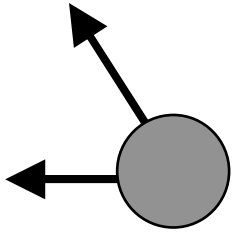
(A)



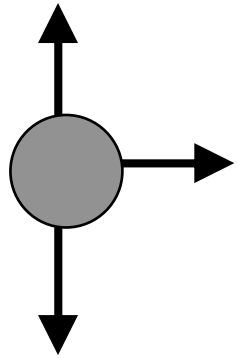
(B)



(C)



(D)



(E)

Problem: Compare the electrostatic force to the gravitational force between the electron and the proton in a hydrogen atom. The distance between the electron and proton is about 5.29×10^{-11} m.

We will need to use $q_e = q_P = 1.6\times10^{-19}$ C, $m_e = 9.11\times10^{-31}$ kg, and $m_P = 1.67\times10^{-27}$ kg. These can all be found on the inside front cover of your book.

Electric Fields

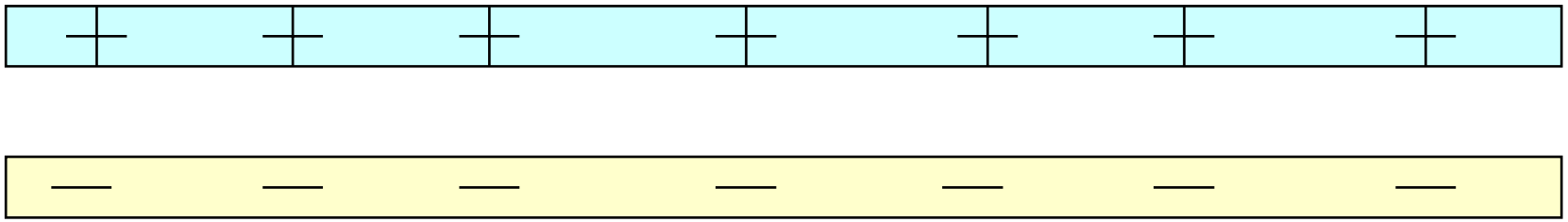
- Fields are a convenient way of describing how one object can affect another object it doesn't touch.
- Electric fields are vectors.
- Electric fields are related to electric forces by the equation, $\mathbf{F} = \mathbf{E}q$, where q is the charge experiencing the force, or the "test" charge.
- A test charge is a charge that doesn't affect the other charges that set up the field.

Interactive Question

Two “point” objects, one with charge q and one with charge Q are separated by a distance r. We know that the magnitude of the electric force of on q from Q is kqQ/r^2. What is the magnitude of the electric *field* experienced by the charge q from Q?

A) kq/r^2
B) kQ/r^2
C) kqQ/r^2

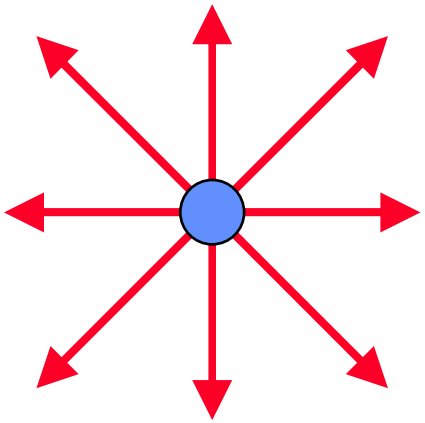
- A uniform electric field is one that is the same over all of space and doesn't change with time.
- A single point charge can not make a uniform electric field. How do you know?
  Because $E = kQ/r^2$ is not the same everywhere. It depends on how far you are from the charge Q.
- A uniform electric field can be made approximately using a parallel plate capacitor. The area between the plates contains a uniform electric field.



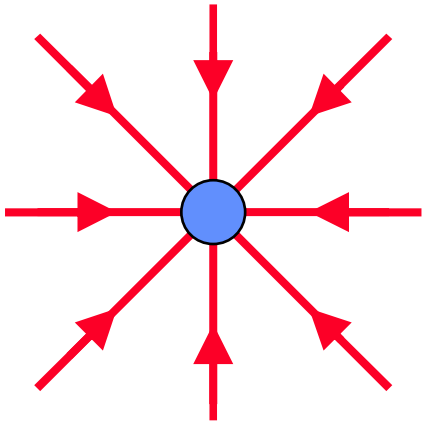
We can illustrate electric fields using electric field lines having the following properties.

1. They point in the direction that a positive charge would move.
2. Their density is proportional to the magnitude of the electric field.
3. Field lines must begin and end on charged particles. (Sometimes not visible in the picture.)
4. The number of field lines beginning or ending on an object is proportional to the magnitude of the charge of the object.

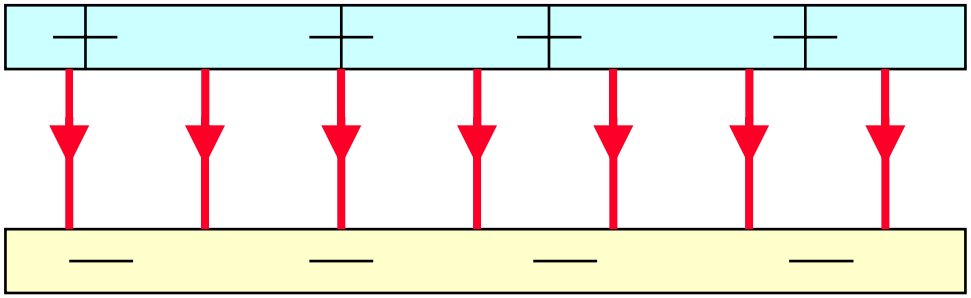
Examples of Electric Field Lines



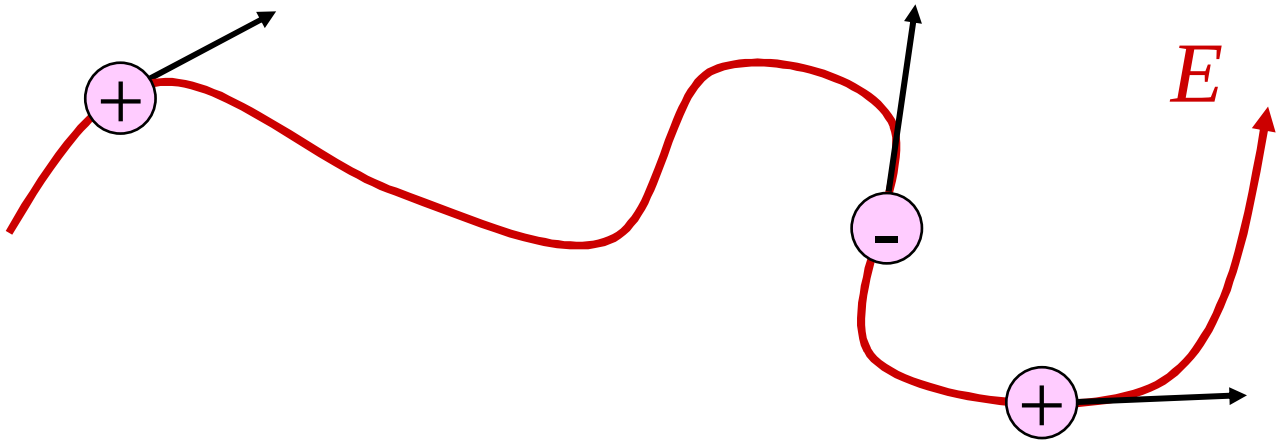
Positive Charge



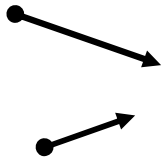
Negative Charge



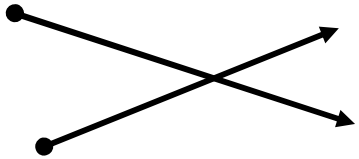
Parallel Plate (Uniform Field)


E

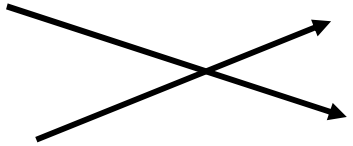
Interactive Question



(1)



(2)

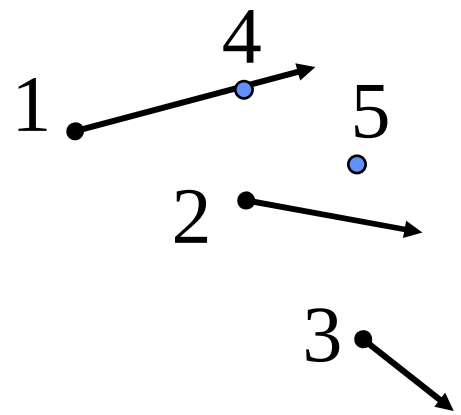


(3)

Pictures (1) and (2) show the electric field at two single points. Picture (3) shows electric field lines. Which of the pictures show an impossible situation?

A) (1) only
B) (2) only
C) (3) only
D) (2) and (3)
E) None of the above

Interactive Question

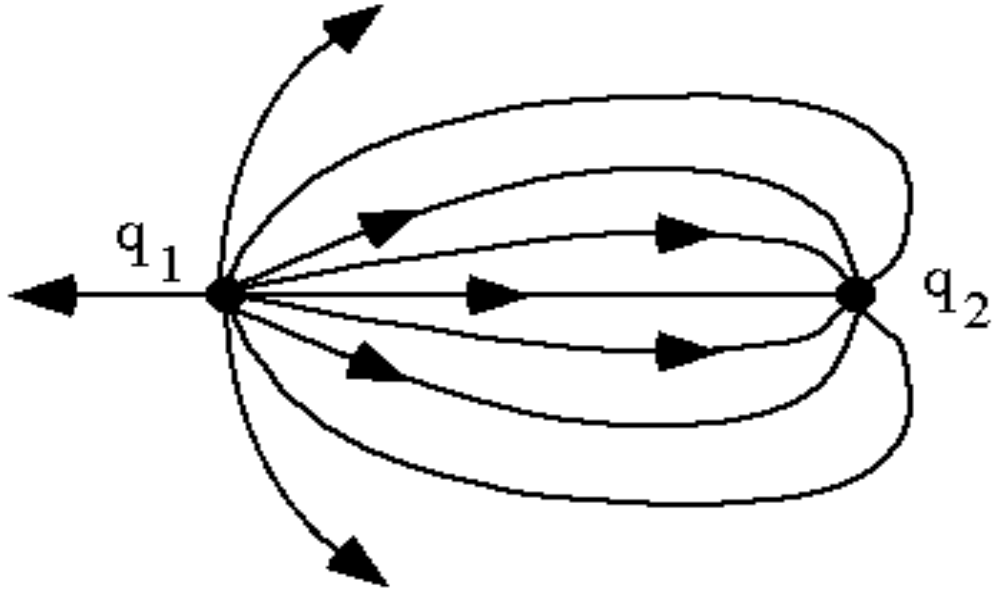


The electric field at three points (1, 2, and 3) is shown. Which of the following is necessarily true?

A) The electric field at point 4 is the same as the electric field at point 1.
B) The electric field at point 5 points mostly to the right.
C) The electric field at point 5 is zero.
D) More than one of the above is true.
E) None of the above is correct

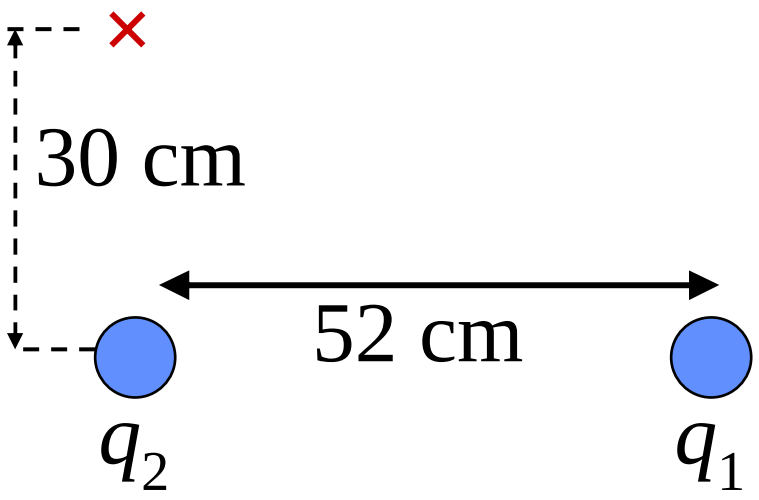
Interactive Question

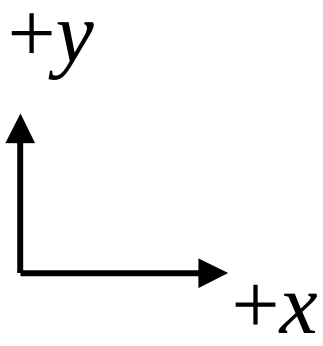
The figure at the right shows the electric field lines in the vicinity of two point charges. Which statement is true?



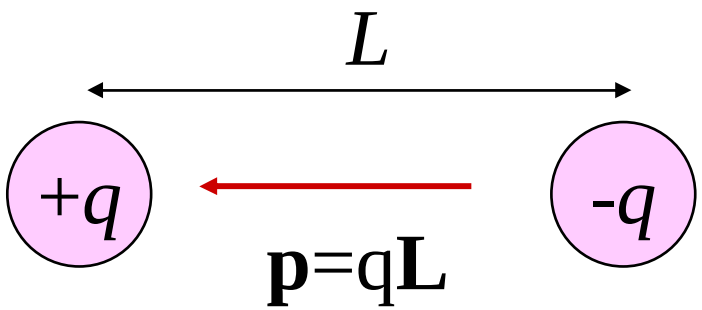
A) q_1 is negative and q_2 is positive
B) The magnitude of the ratio of q_2/q_1 is 0.70
C) Both q_1 and q_2 have the same sign of charge
D) The strength of the electric field is the same everywhere
E) The electric field is the strongest midway between the charges.

Problem: (a) Find the electric field at a point located 30 cm from q_2 along the $+y$ axis if $q_1 = -50\times10^{-6}$ C and $q_2 = 50\times10^{-6}$ C?
(b) What is force on a particle with a charge of Q at that point?

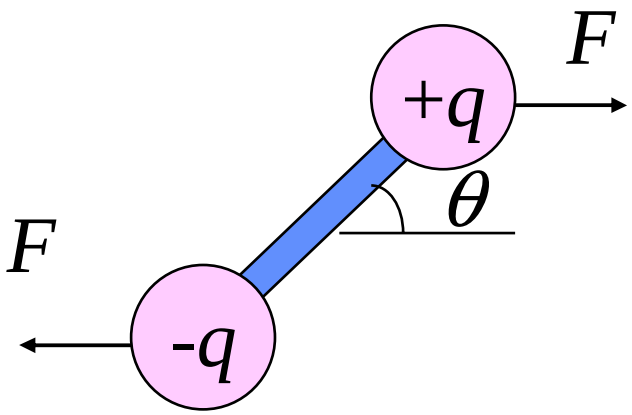




Electric Dipoles



p: The electric dipole moment



$\boldsymbol{\tau} = 2(\mathbf{L}/2)\times\mathbf{F}$

$= \mathbf{L}\times q\mathbf{E} = q\mathbf{L}\times\mathbf{E}$

$= \mathbf{p}\times\mathbf{E}$

$U = -W$

$= -\int(-\tau\, d\theta)$

(–because τ decreases θ)

$= \int pE \sin\theta\, d\theta$

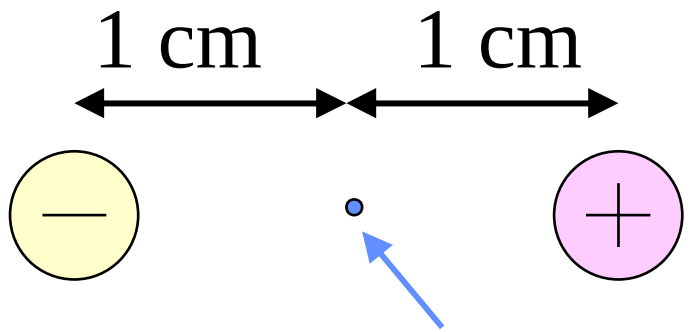
$= -pE\cos\theta_f + C$

Chose U=0 when θ=90°

$U = -pE\cos\theta$

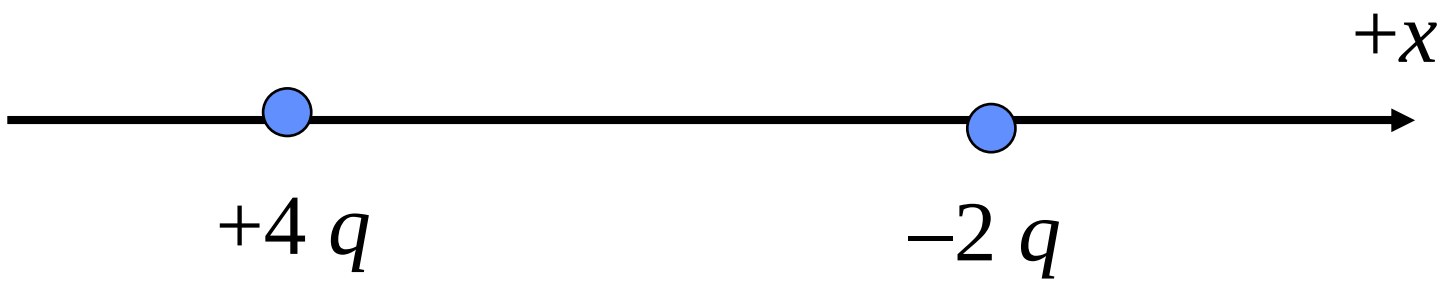
$U = -\mathbf{p}\cdot\mathbf{E}$

Problem: Two metal spheres are place 2 cm apart. Equal amounts of charge are put on each sphere (positive on one and negative on the other) until a spark occurs between the two spheres. How much charge is on each sphere just before it sparks? (Hint: A spark occurs when the strength of the electric field between the two spheres equals the dielectric strength of air. The dielectric strength of air is 3.0×10^{6} N/C (or V/m).)



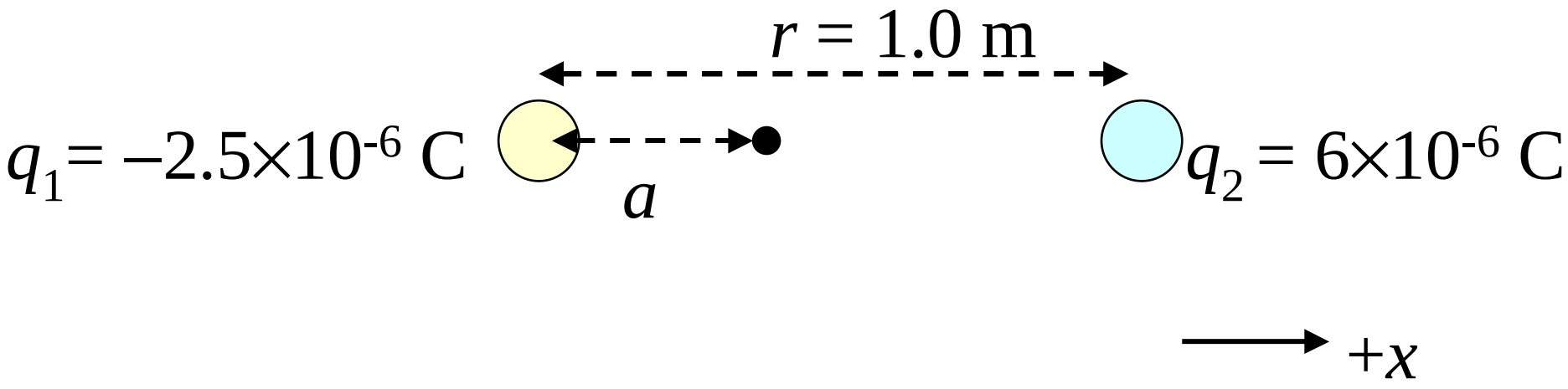
Interactive Question

At which point is the electric field zero for the two point charges shown?



A) It is never 0.
B) Somewhere to the left of the +4q charge.
C) Somewhere to the right of the –2q charge.
D) Somewhere between the two charges, but nearer to the –2q charge.
E) To the right of the –2q charge and the left of the +4q charge.

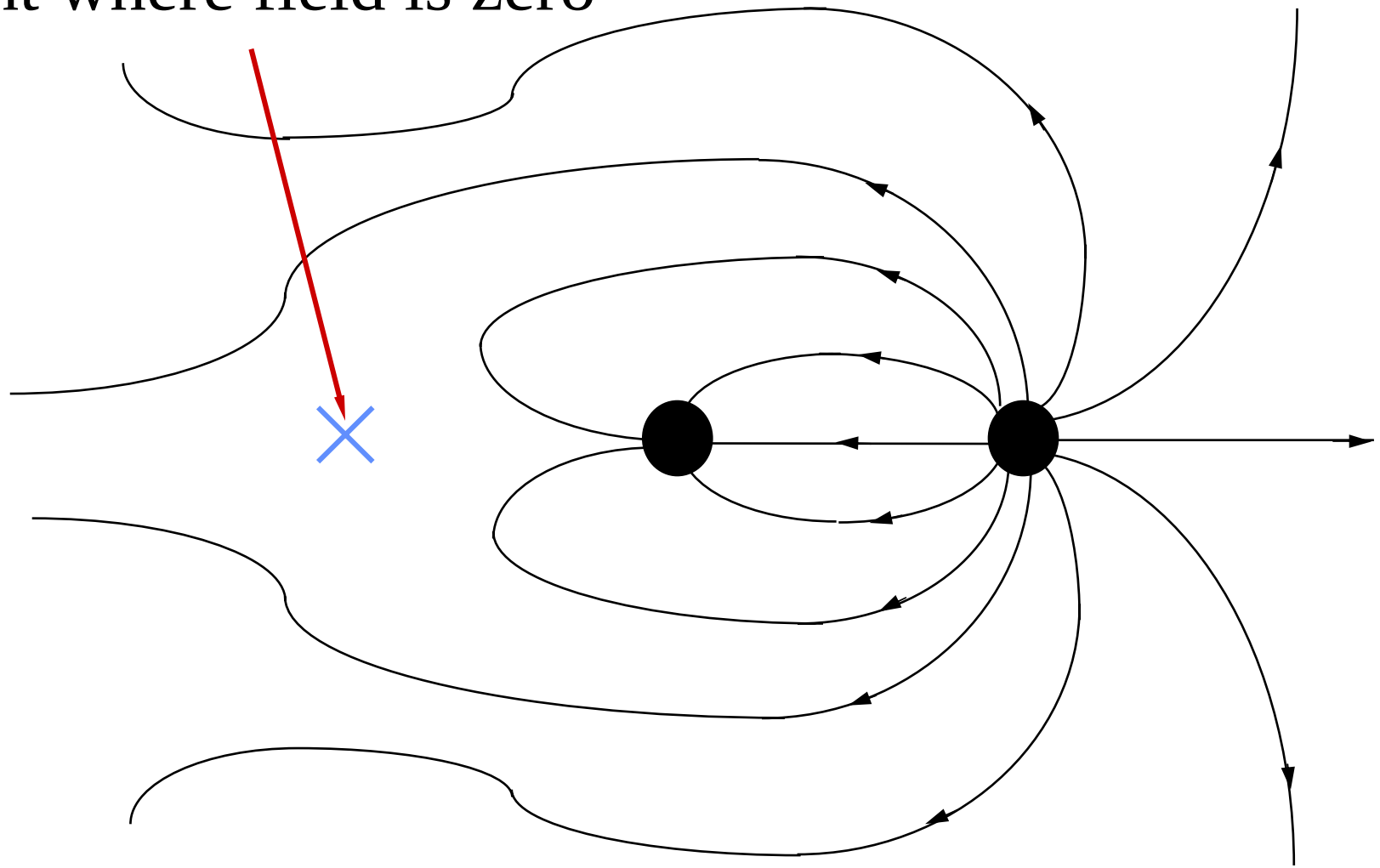
Problem: Where is electric field equal to zero along the *x* axis in this figure? Remember, you **must** use magnitudes (absolute values of the two charges) to solve problems with Coulombs law.



a is the point where the field is zero. It could be a positive or negative number.

The electric field lines would look something like this.

Point where field is zero



The movement of particles in electric fields

We know that a net force causes an acceleration on an object. Let's look at Newton's second law when the only force on an object is the electric force.

$\sum F = ma$

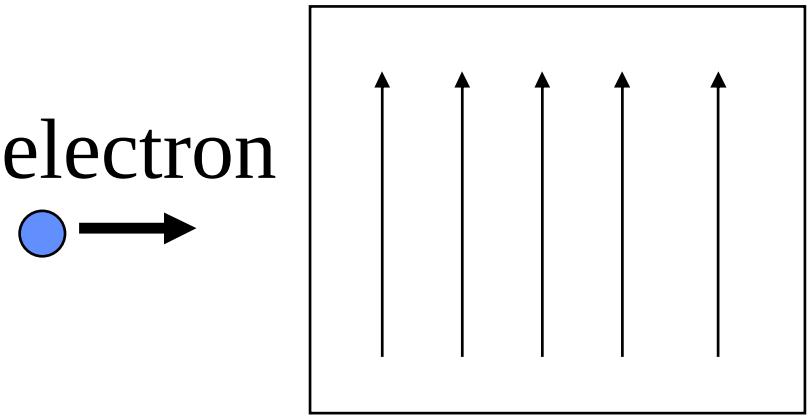
$Eq = ma$

How do each of the following affect the acceleration?

1. The sign of the charge?
2. The magnitude of the charge?
3. The mass of the object?
4. The electric field strength?
5. A particle with no electric charge?

Interactive Question

An electron traveling horizontally enters a region where a uniform electric field is directed vertically upward. What is the direction of the force exerted on the electron once it has entered the field?

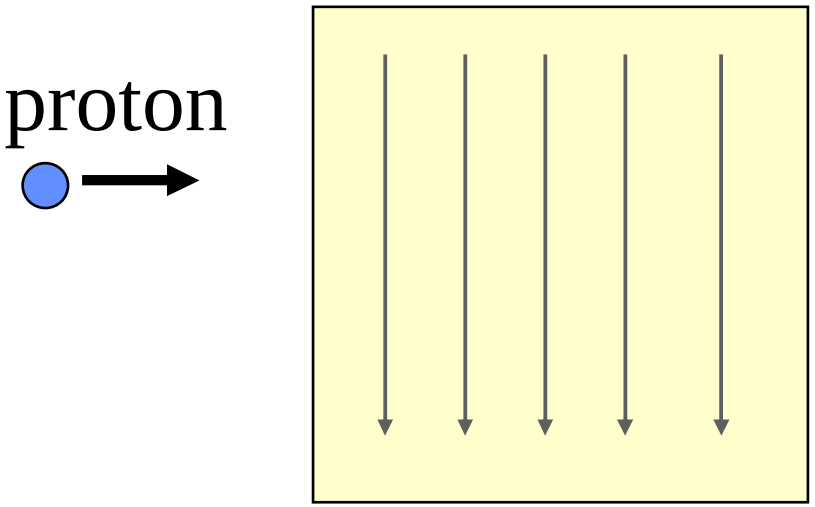


A) upward
B) to the left
C) to the right
D) downward
E) out of the page toward the reader.

Interactive Question

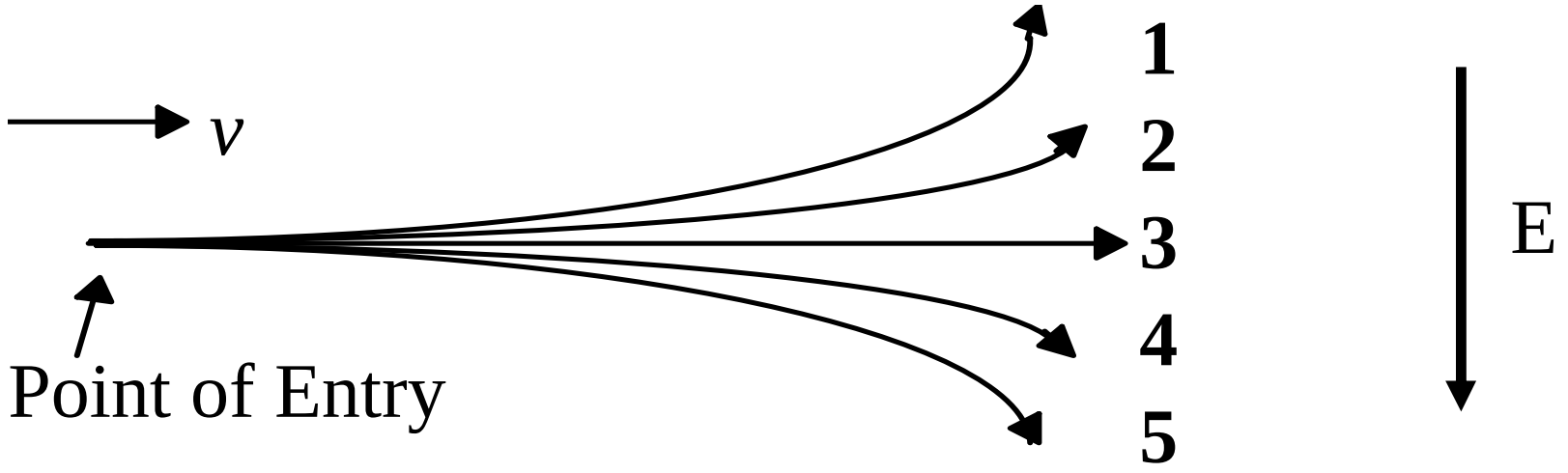
A proton traveling horizontally enters a region where a uniform electric field is directed vertically downward. The proton will move downward. What will be the shape of the proton's path while it is in the uniform electric field?

A) a straight line
B) a circle
C) a parabola
D) a hyperbola
E) none of the above



Interactive Question

Two particles of the same mass carry charges +3Q and –2Q respectively. They are shot into a region that contains a uniform electric field. The lines below show possible paths for the particles. If the electric field points down, what will be the paths for these particles?



A) **1** for +3Q, **4** for –2Q

B) **3** for +3Q, **2** for –2Q

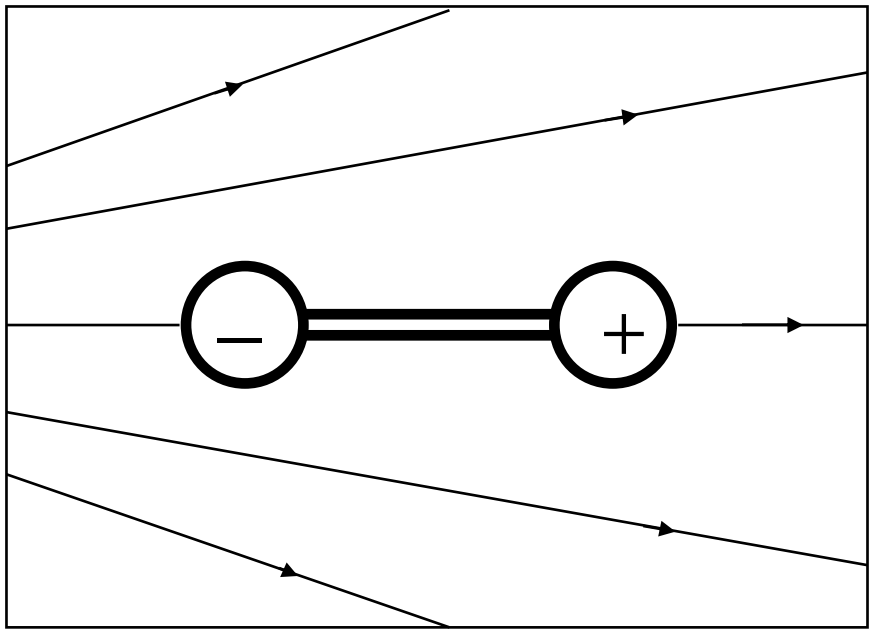
C) **4** for +3Q, **3** for –2Q

D) **2** for +3Q, **5** for –2Q

E) **5** for +3Q, **2** for –2Q

Interactive Question

An electrically neutral dipole is placed in a nonuniform external field shown by the arrows. If the dipole is initially at rest, which direction will it move?

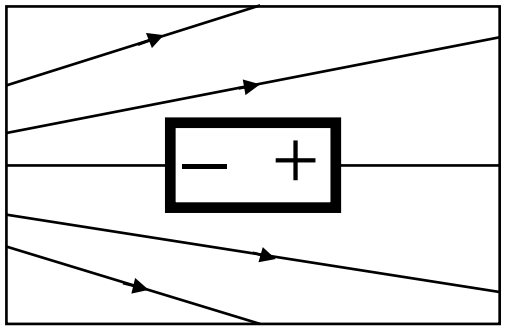


A) to the left
B) to the right
C) upward, to the top of the page
D) downward, to the bottom of the page
E) does not move at all

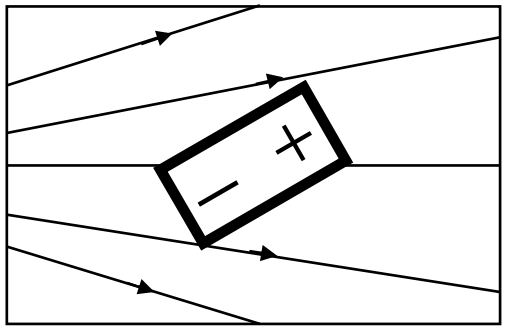
Interactive Question

An electrically neutral dipole is placed in an external field shown by the arrows below. In which situations(s) is the net force on the dipole zero?

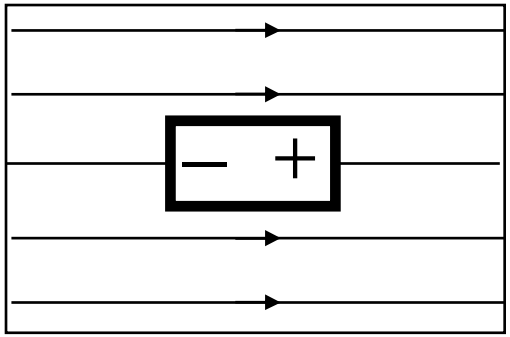
A) 1
B) 3
C) 2 & 4
D) 1 & 3
E) 3 & 4



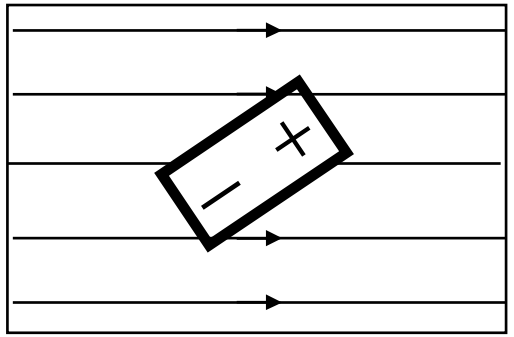
(1)



(2)



(3)

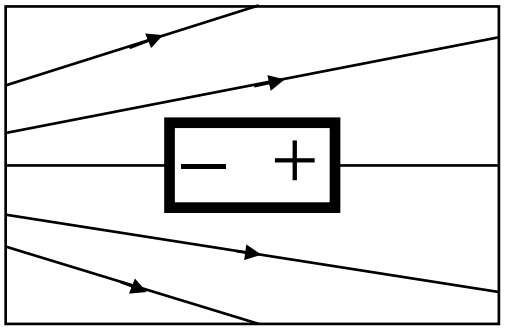


(4)

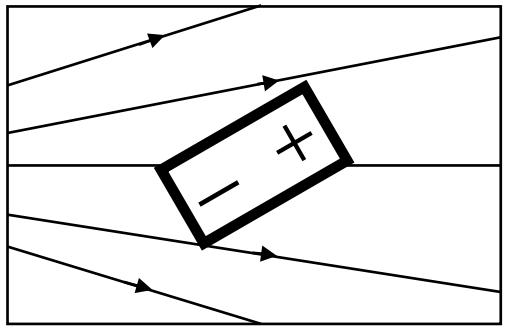
Interactive Question

An electrically neutral dipole is placed in an external field show by the arrows below. In which situations(s) is the net torque on the dipole zero?

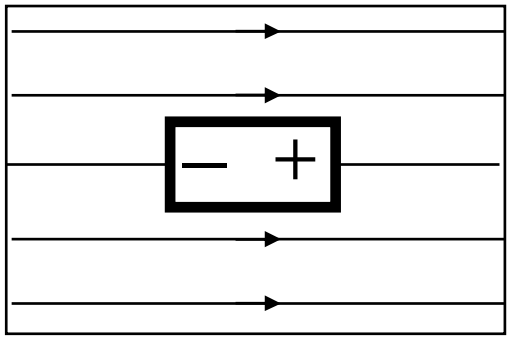
A) 3
B) 4
C) 2 & 4
D) 1 & 3
E) 3 & 4



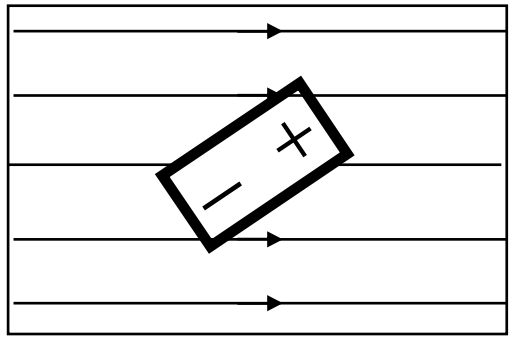
(1)



(2)



(3)



(4)

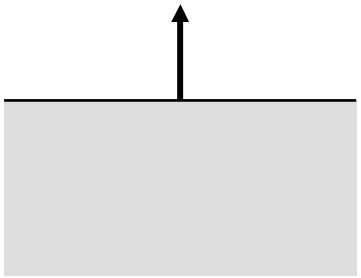
Electric Field and Charge In a Conductor

If there is an electric field inside a conductor, the charged objects (electrons) will feel a force and move freely. As they move, the field changes, since the electrons change position. They will move until there is no more force, which means there is no more field. Some consequences of this *for the static case* (once the electrons stop moving) are:

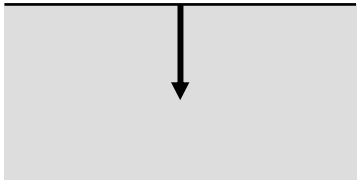
1. There is no electric field inside a conductor.
2. All charge on a conductor is located on the surface.
3. All field lines are directed perpendicular to the surface

Interactive Question

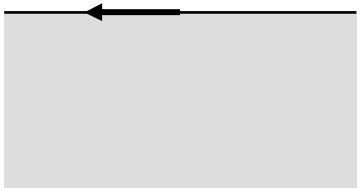
In the electrostatic case, what direction could the electric field point near the surface of a conductor?



A)



B)



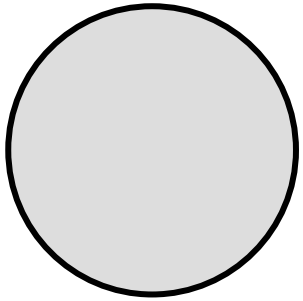
C)

D) More than one of the above

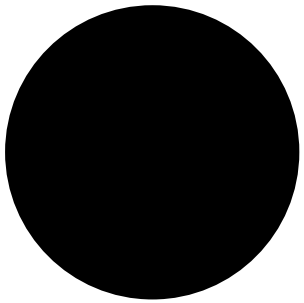
E) None of the above

Interactive Question

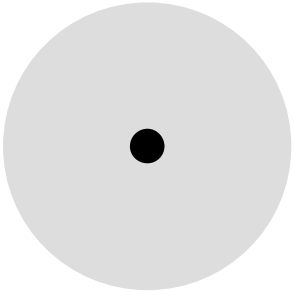
A solid spherical conductor has a net charge of 10 mC on it. In the electrostatic situation, where would you expect to find the charge (shown by dark bands)? Cross sections of the sphere are shown.



A)



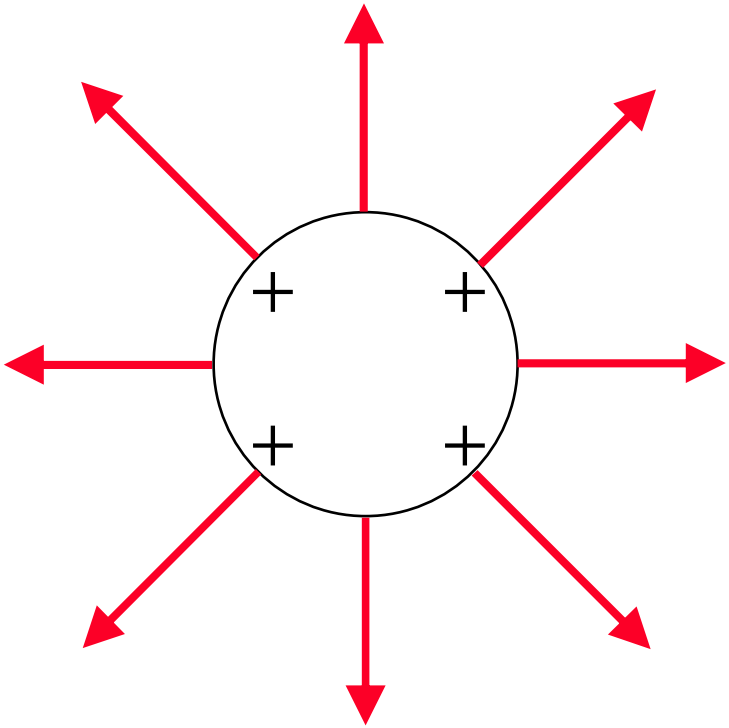
B)



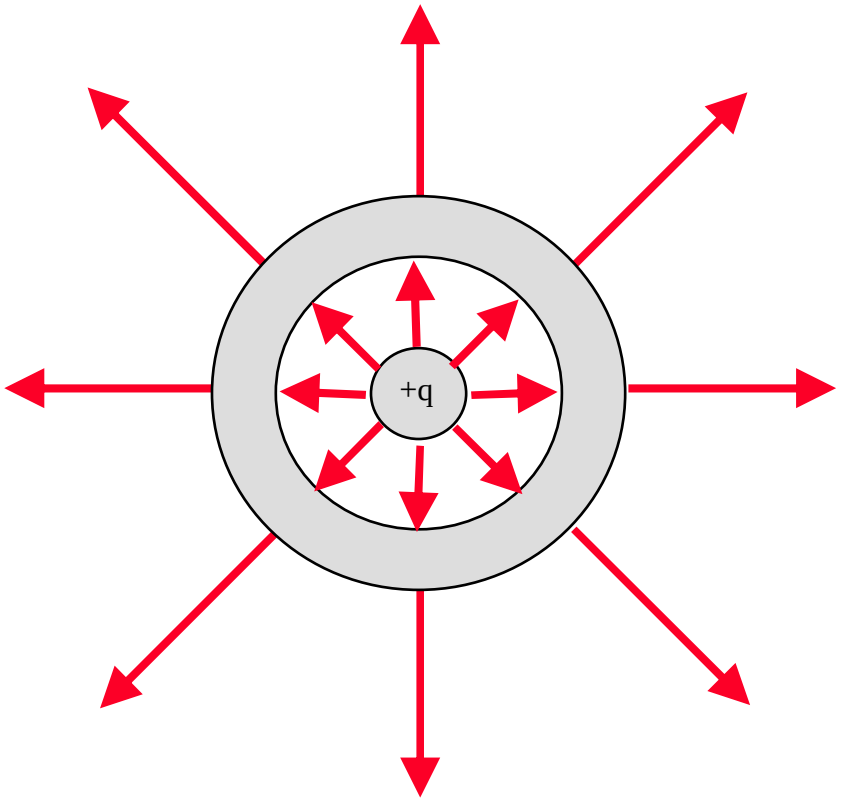
C)

D) More than one of the above

E) None of the above



Charged sphere

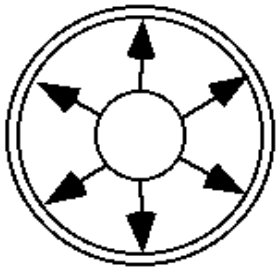


Charged sphere inside
A hollow neutral sphere

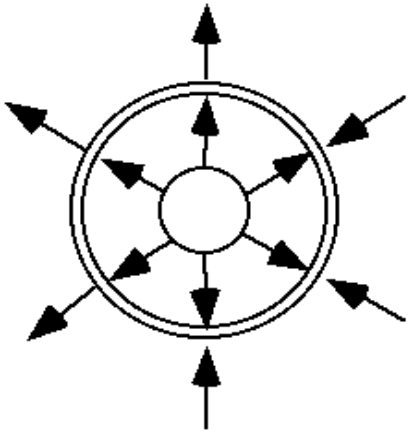
Interactive Question

A solid inner sphere carries a charge of +6 μC. Concentric with this sphere is a conducting hollow sphere which also carries a charge of +6 μC. Which figure accurately shows the field lines?

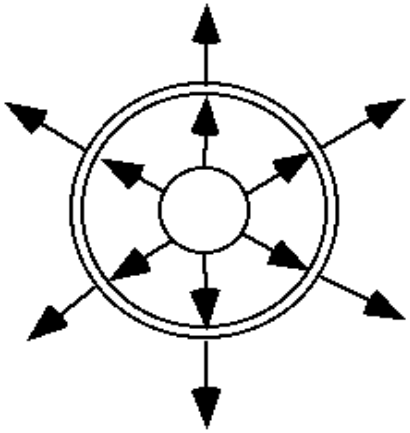
A)



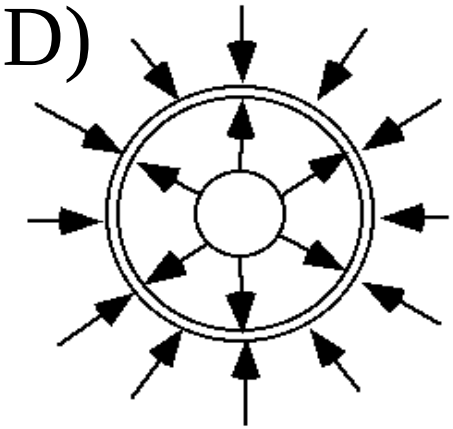
B)



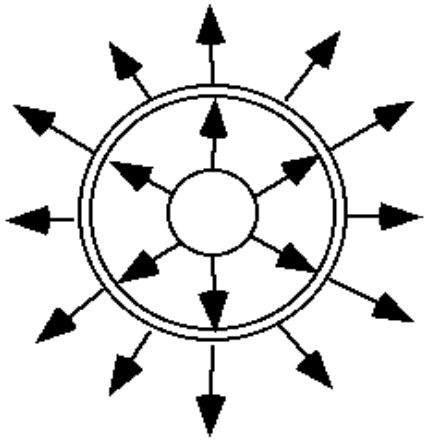
C)



D)

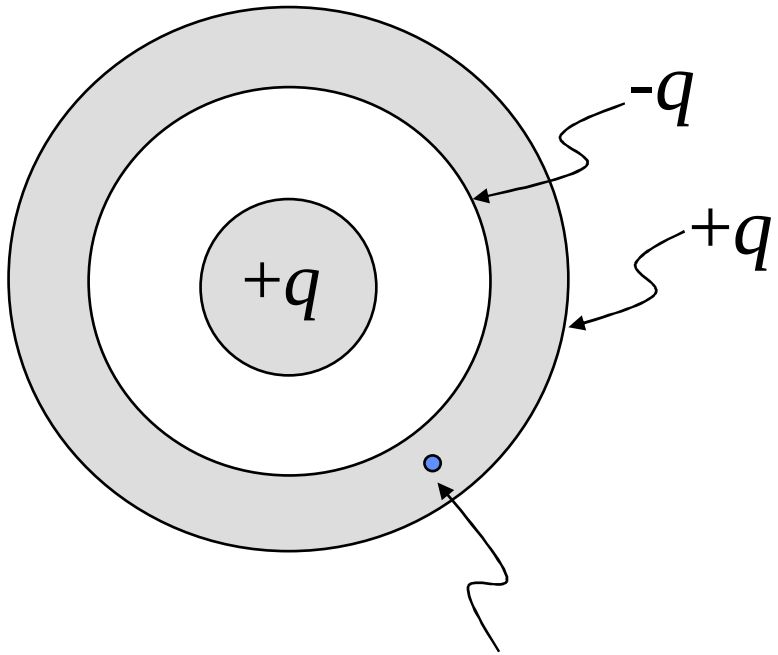


E)



Interactive Question

A conducting sphere with a charge of $+q$ is placed inside another hollow conducting sphere. Initially equal and opposite charges are on the inner and outer surfaces of the hollow sphere. Electrons are removed from the outer conductor so it now has a net positive charge. What will happen to the positive charge?



A) It will not change location.
B) It will move to the inner surface of the conductor.
C) It will move to the outer surface of the conductor.

The Electric Field of Continuous Charge Distributions

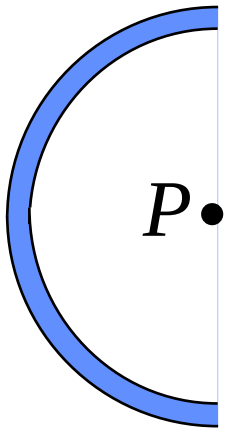
Up until now, we have calculated the electric field from a distribution of point charges. Now we will learn two different methods to calculate the static electric field from a distribution of charge.

1. Integration over an infinitesimal charge distribution
2. Gauss's Law

In theory, both can be used in any circumstances. However, Gauss's law is only practical to use when there is are certain obvious symmetries in the problem. When Gauss's law can be used it is almost much simpler than the integration method.

Interactive Question

Positive charge Q is uniformly distributed on a semicircular rod. What is the direction of the electric field at point P, the center of the semicircle?



A) ↑

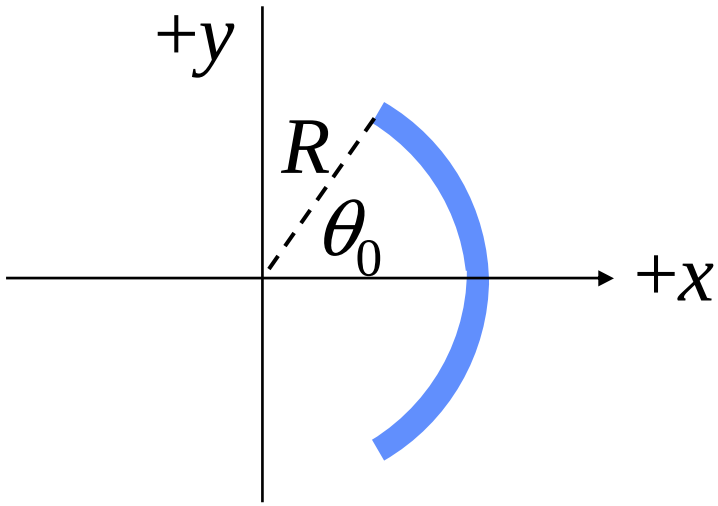
B) ↓

C) ←

D) →

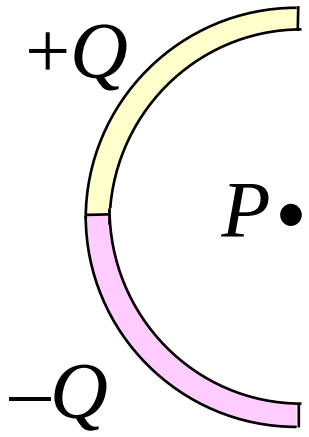
E) ↗

Problem: A circular arc that subtends an angle θ_0 is placed symmetrically on the x axis as shown. There is a total charge on the arc of q. What is the electric field at the center of the circle, a distance R from the arc?



Interactive Question

Positive charge $+Q$ is uniformly distributed on the upper half of a semicircular rod and negative charge $-Q$ is uniformly distributed on the lower half.. What is the direction of the electric field at point P, the center of the semicircle?



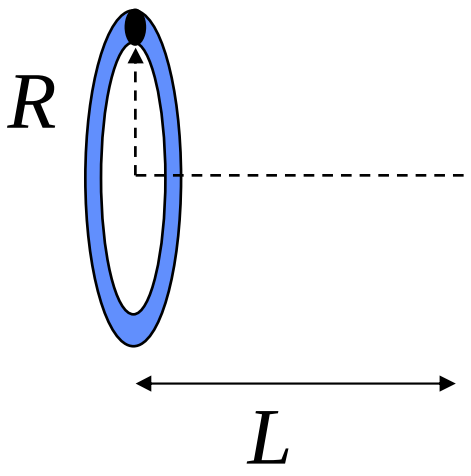
A) ↑ B) ↓ C) ← D) → E) ↗

Solving Problems with Continuous Charge Distributions

1. Identify type of charge distribution:
   Linear: $dq = \lambda\, ds$
   Surface: $dq = \sigma\, dA$
   Volume: $dq = \rho\, dV$
2. Draw a picture of the problem, including dq, the vector $\mathbf{r}$ and the contribution $d\mathbf{E}$ arising from dq at the point to be evaluated, P.
3. Look for something that will simplify the calculation:
   Is the charge distribution a *sum* of ones with a known solution?
   Does *symmetry* require that some component of the field be zero?

4. Write down an expression for the magnitude of $d\mathbf{E}$.
   If it is a small point-charge contribution then:
   $$dE = K\, dq/r^2$$
   If you are using a previous result then:
   $$E = E_{\text{prev}}(dq)$$
5. Resolve $d\mathbf{E}$ into components, dE_x, dE_y, and dE_z (ignore those that will later sum to zero).
6. Determine the variable over which you will integrate. **(Note:** Only integrate over region where the charge distribution exists!) Write your expression for the components of $d\mathbf{E}$ in terms of this variable.
7. Do the integral.
8. Sum the components: $\mathbf{E} = E_x\mathbf{i} + E_y\mathbf{j} + E_z\mathbf{k}$.
9. Check to see if your answer agrees with your intuition.

Problem: What is the electric field along the axis of a ring of total charge Q of radius R?

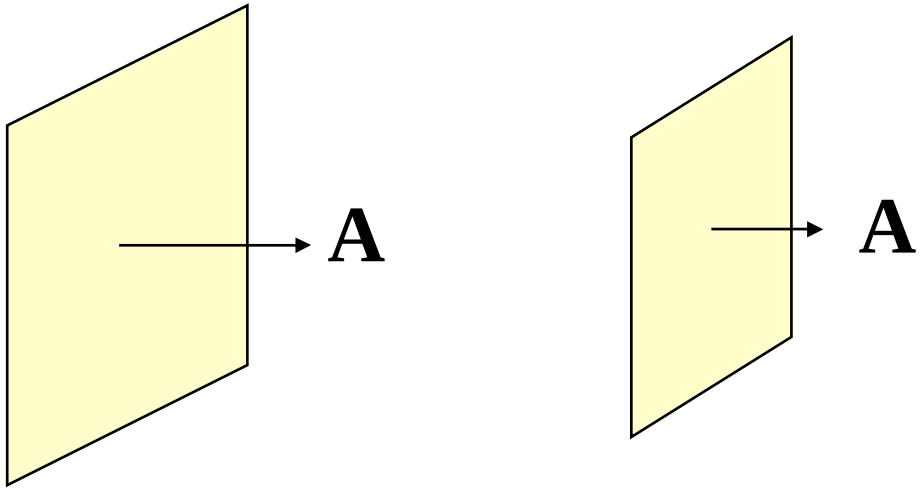


Problem: What is the electric field of a thin disk with a total charge Q_0 and a radius R_0?

Electric Flux

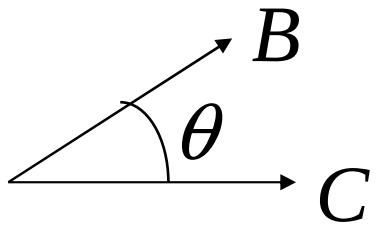
Electric Flux is the total electric field passing through a surface.

$\Phi_E = \int \mathbf{E} \cdot d\mathbf{A}$



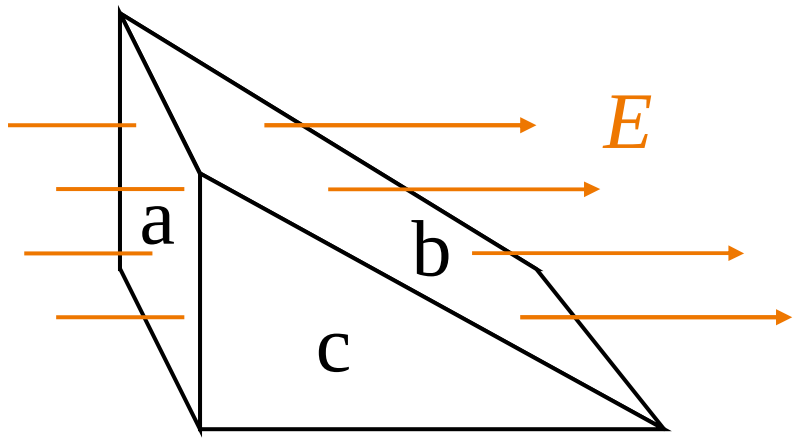
If the field is uniform and its direction doesn't change relative to the surface, then $\Phi_E = \mathbf{E} \cdot \mathbf{A}$

$$\mathbf{B} \cdot \mathbf{C} = BC\cos\theta = B_x C_x + B_y C_y + B_z C_z$$



Interactive Question

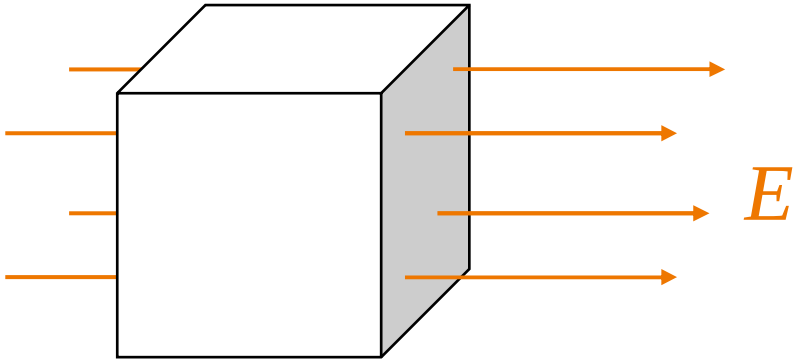
The closed triangular surface shown to the left is placed in a uniform electric field as shown. Which side(s) of the object has the greatest electric flux passing through it?



A) Side (a)
B) Side (b)
C) Side (c)
D) Side (a) and (b)
E) Side (a), (b), and (c) are all equal

Interactive Question

Consider the closed surface shown below, with each side being 0.1 meters. The arrows represent a constant electric field of 1 N/C. What is the electric flux through this surface?



A) 0.0 $N \cdot m^2/C$

B) 0.01 $N \cdot m^2/C$

C) 0.02 $N \cdot m^2/C$

D) 0.06 $N \cdot m^2/C$

E) 0.2 $N \cdot m^2/C$

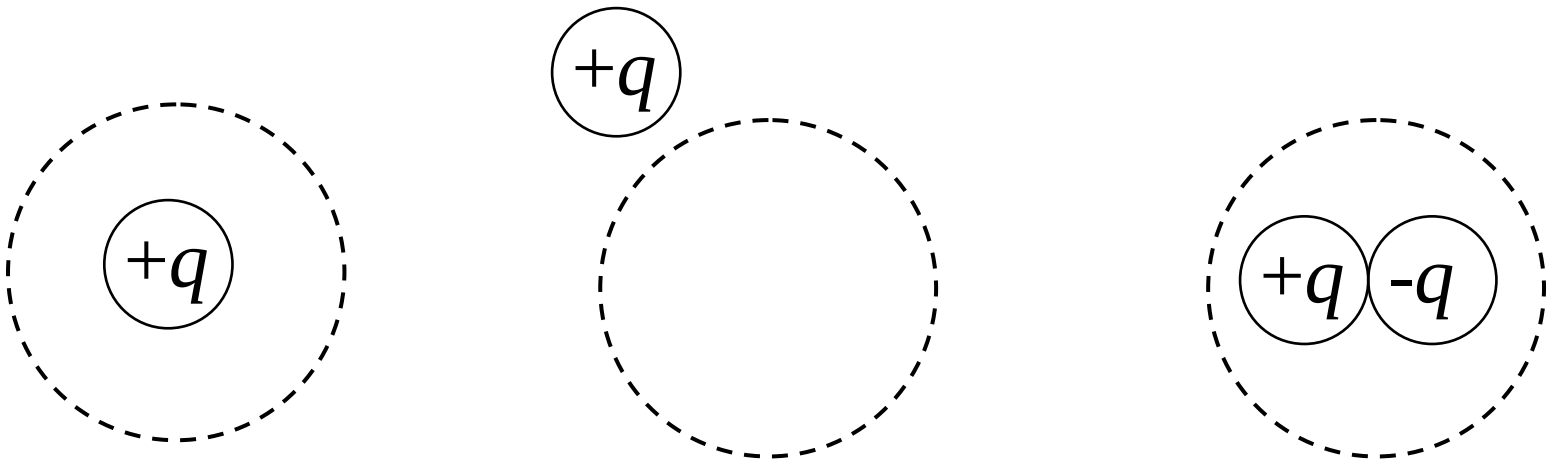
Gauss's Law

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = q_{\text{encl}}/\varepsilon_0$$

- The integral is the integral around *any* closed surface.
- The q_{encl} is the charge *inside* that same surface.
- The **E** is the electric field going through the surface due to all sources of electric field.
- Gauss's law applies to all circumstances, but is easiest to use when there is an obviously symmetric situation.

Interactive Question

Which Gaussian surface shown below by the dotted lines has a total flux through it equal to zero. (Consider each situation separately)



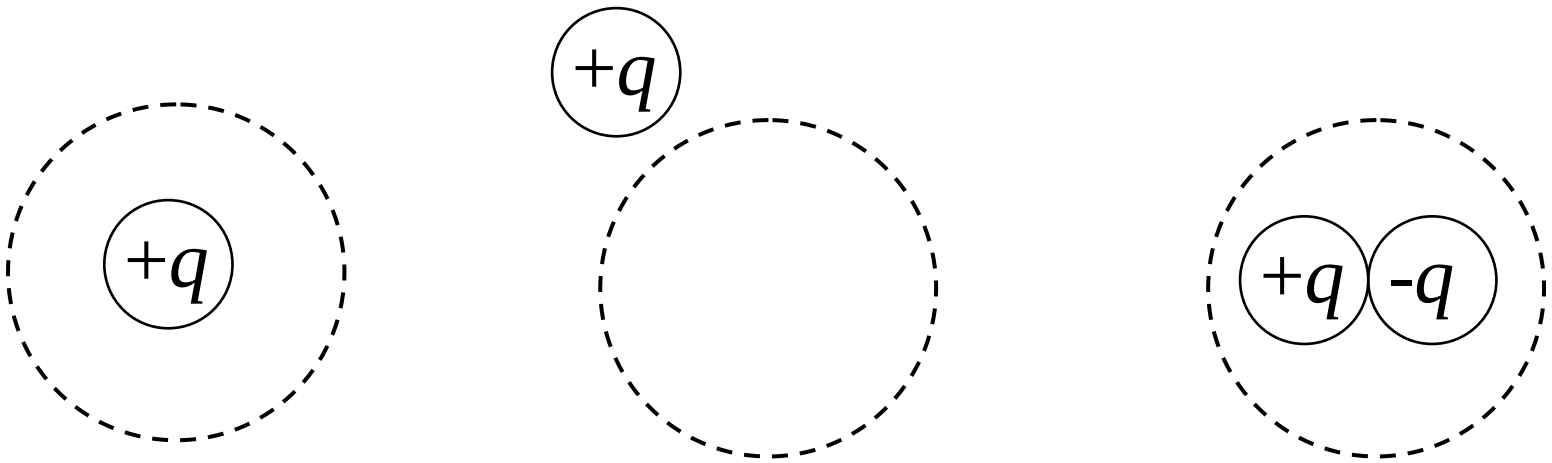
A) B) C)

D) More than one of the above.

E) None of the above.

Interactive Question

On which Gaussian surface shown below will the electric field be zero at each point on the entire surface?

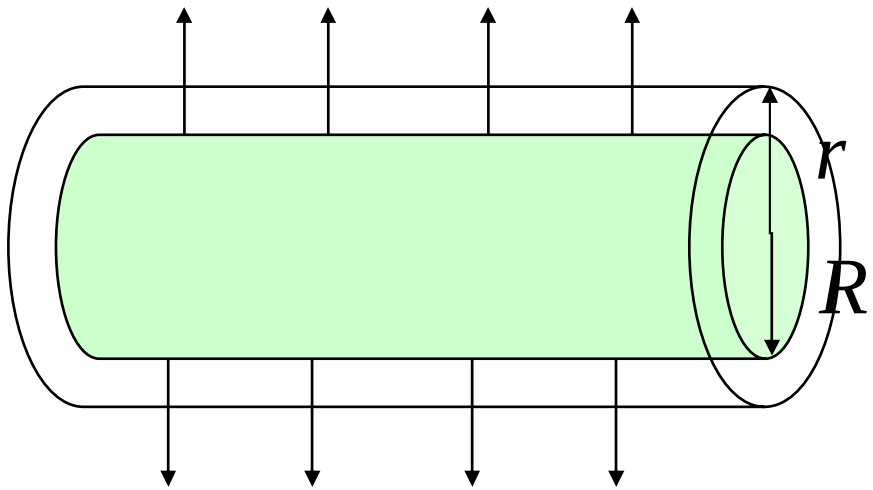


A) B) C)

D) More than one of the above.

E) None of the above.

Problem: A long, hollow cylindrical conductor with radius R has a positive charge per unit area given by σ along its entire surface. What is the electric field inside and outside of the conductor?

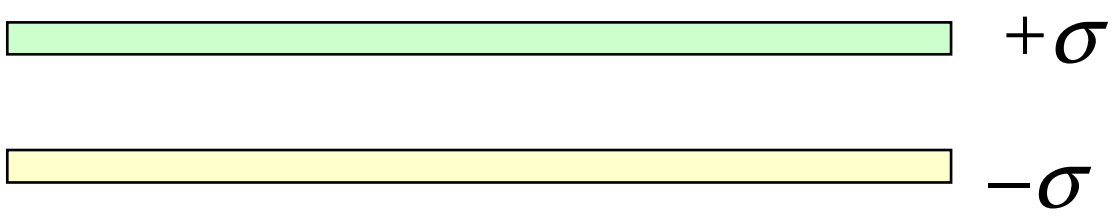


Problem: What is the charge inside any conductor?

Problem: What is the electric field on the surface of any conductor? The conductor has a charge per unit area of σ.

Interactive Question

The electric field for an infinite plate with charge per unit area of σ is σ/ε_0. When one plate with charge per unit area of σ is placed parallel to another plate with charge per unit area of $-\sigma$, the magnitude of the electric field is:

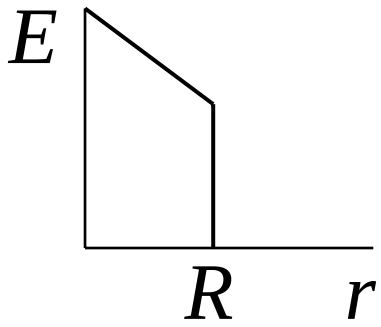


A) $2\sigma/\varepsilon_0$ between the plates and 0 outside.
B) $2\sigma/\varepsilon_0$ between the plates and $\pm\sigma/\varepsilon_0$ outside.
C) Zero between and outside the plate.
D) $\pm\sigma/\varepsilon_0$ both between and outside the plates.
E) None of the above.

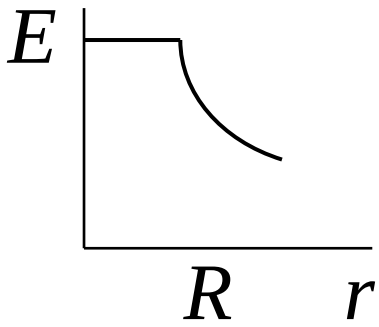
Problem: A nonconducting sphere of radius R has a uniform charge distribution distributed throughout the sphere. The total charge on the sphere is Q. What is the electric field inside the sphere and outside the sphere?

Interactive Question

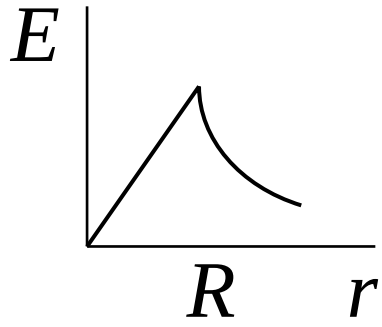
Which graph represents the magnitude of the electric field as a function of the distance from the center of a solid, charged nonconducting sphere of radius *R*?



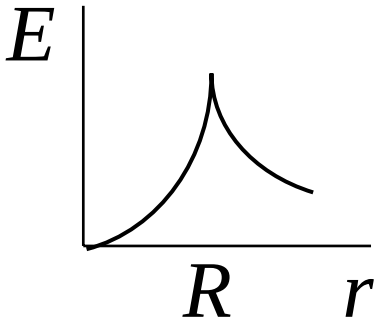
(A)



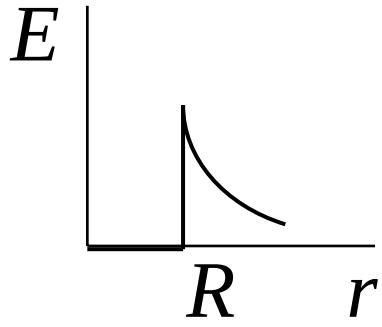
(B)



(C)



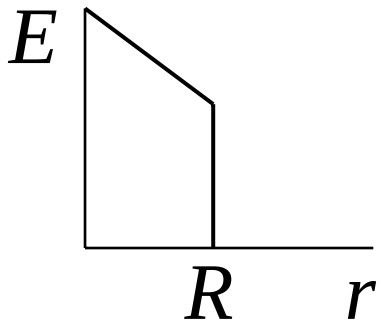
(D)



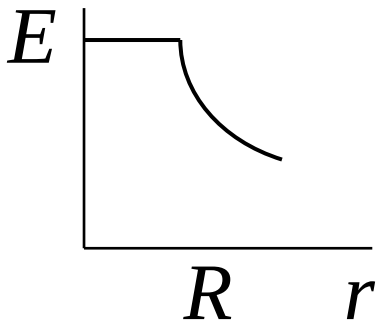
(E)

Interactive Question

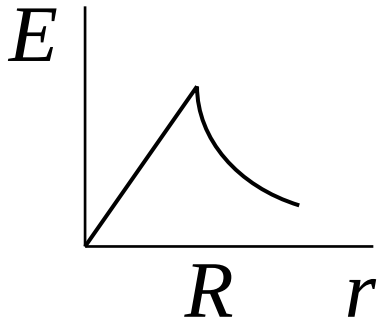
Which graph represents the magnitude of the electric field as a function of the distance from the center of a solid, charged *conducting* sphere of radius *R*?



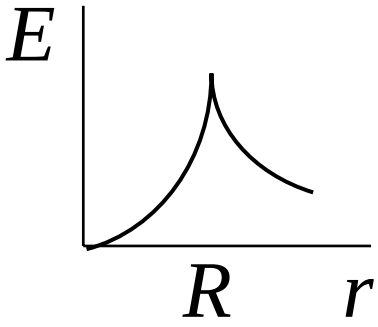
(A)



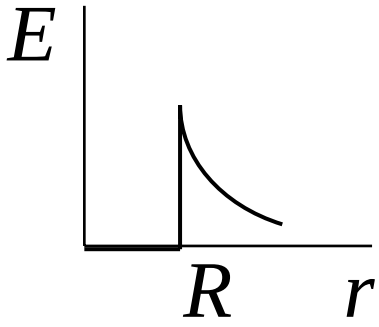
(B)



(C)

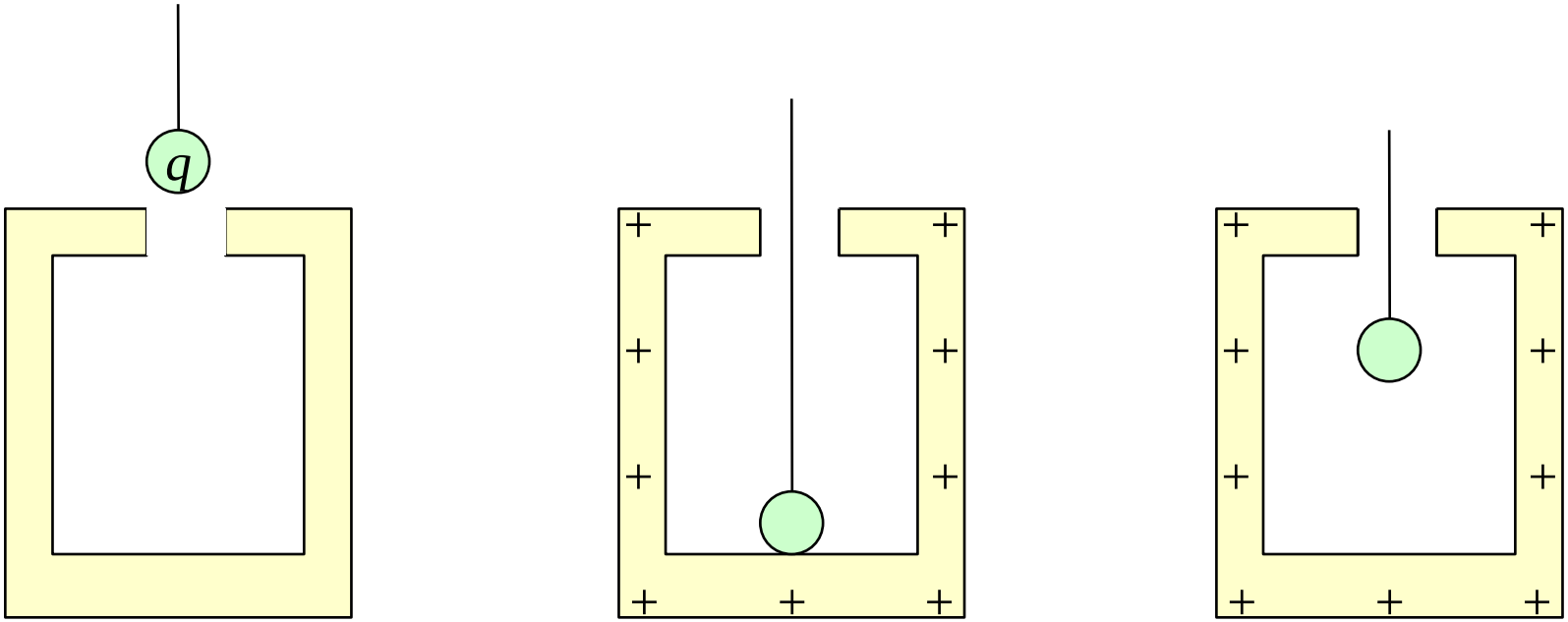


(D)



(E)

Problem: A nonconducting sphere with a radius of R has a volume charge density given by Ar^2. What is the electric field inside and outside the sphere?

Problem: An infinitely long cylinder has a charge density given by $\rho = ar$, where r is the distance from the axis of the cylinder. Find the electric field (a) inside and (b) outside of this cylinder.

Verifying Gauss's Law