Red Hot Half-Life
Modeling Nuclear Decay

About this Lesson
This lesson can be used in multiple places within a chemistry curriculum. It can be used with the atomic structure unit, a nuclear chemistry unit or a kinetics unit. The lesson begins with a hands-on activity where students model nuclear decay and concludes by analyzing the data with a graphing calculator to determine the half-life of the cinnamon candies. The exponential function determined from the graph is transformed into a linear function and the half-life is then calculated from the slope of this line for comparison.

This lesson is included in the LTF Chemistry Module 3.

Objectives
Students will:

- Model a system of nuclear decay using cinnamon candies.
- Analyze the data using a graphing calculator.

Level
Chemistry

Common Core State Standards for Science Content
LTF Science lessons will be aligned with the next generation of multi-state science standards that are currently in development. These standards are said to be developed around the anchor document, *A Framework for K–12 Science Education*, which was produced by the National Research Council. Where applicable, the LTF Science lessons are also aligned to the Common Core Standards for Mathematical Content as well as the Common Core Literacy Standards for Science and Technical Subjects.

Code	Standard	Level of Thinking	Depth of Knowledge
(LITERACY) RST.9-10.3	Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.	Apply	II
(MATH) A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.	Apply	II
(MATH) A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	Apply	II
(LITERACY) RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into	Apply	II

Code	Standard	Level of Thinking	Depth of Knowledge
	words.		
(MATH) S-ID.6a	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.	Apply	II
(MATH) S-ID.6b	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Informally assess the fit of a function by plotting and analyzing residuals.	Apply	II
(MATH) S-ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit.	Apply	II
(MATH) S-ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	Apply	II
(MATH) F-LE.5	Interpret expressions for functions in terms of the situation they model. Interpret the parameters in a linear or exponential function in terms of a context.	Apply	II

Connections to AP*

AP Chemistry:

I. Structure of Matter A. Atomic Theory and Atomic Structure 4. Electron energy levels; atomic spectra, quantum numbers, atomic orbitals

AP Physics:

graphing and transformation of equations

**Advanced Placement and AP are registered trademarks of the College Entrance Examination Board. The College Board was not involved in the production of this product.*

Materials and Resources

Each lab group will need the following:

beans, navy
calculator, TI® graphing
cinnamon candies
2 cups, 9-oz clear plastic
paper towels
plate, paper

Additional teacher materials:

marker, Sharpie®

Assessments
The following types of formative assessments are embedded in this lesson:

- Assessment of prior knowledge.
- Graphing of exponential functions and linearizing data.
- Guided questions during the activity to assess conceptual understanding of half-life.
- Comparison of group data to determine differences in half-life.

The following additional assessments are located on the LTF website:

- Chemistry Assessment: Structure of Matter
- AP Style Free Response

Teaching Suggestions
Before class, use a marker to divide each paper plate into equal sections. Some plates should be divided in thirds, some in fourths, some in sixths, some in eighths. Put a star in one of the sections. Each group should have a sample of candies containing approximately 150−180 pieces and a similar size sample of navy beans. Put the candy and beans into the paper cups for distribution. Depending upon your own lab procedure, you may allow students to eat the candy after the lab is complete.

The students count the number of red hots in the original sample and record. They then pour the candies onto the paper plate and remove the candy that falls into the starred section. The students count the remaining candies and record. Have the students replace the "decayed" candies with navy beans. By replacing the decayed candies students will realize that the size of the sample does not change – it only becomes less radioactive with each half life. Each trial is considered to be 10 seconds. The procedure is continued for ten trials or until the number of candies is reduced to less than 5, whichever comes first.

Students will analyze the data collected with a graphing calculator. You may refer to *Foundation Lesson VI: Use of the Graphing Calculator* for help. While this lesson has been designed as a calculator lesson, it may also be done as a paper/pencil graphing exercise with simple modifications.

Answer Key

Data and Observations

Start with 180 candies and a plate divided into 6 sections.

Table 1. Simulating Radioactive Decay		
Time (s)	**Number of Candies Decayed**	**Number of Candies Remaining**
0	0	180
10	30	150
20	25	125
30	22	103
40	17	86
50	15	71
60	12	59
70	10	49
80	8	41
90	7	34
100	5	29

TEACHER

Answer Key (continued)

Analysis

1. The graph is curved downward.
2. The graphs are all curved downward but some of them are more sharply curved than others.
3. Equations will vary. This is an exponential decay function rather than a linear equation, so you may have to provide a considerable amount of assistance to the students at this point.

   Point students to the form of the equation that is at the top of the screen. You might want to write the example on the board. It will look like $y = ab^x$. The equation for the sample data is

   $$y = (180)(0.98)^x$$

4. Answers will vary depending on the number of sections on the plate. The half-life for the sample data (plate divided into 6 sections) is 37 seconds.

   Because half-life is defined as the time it takes for half of a sample to decay, when the number falls to half the original number the x-coordinate value will be equal to the half-life.

5. When one fourth of the candies are left, it should approximately equal two half-lives.

   Remember that this is real data, so it may not be exact. The sample data shows an x-value of 75 seconds rather than 74 seconds.

6. To perform a linear transform on $y = ab^x$, you will need to take the natural log (ln) of both sides of the equation. You should end up with

   $$\ln y = \ln(a) + (x)[\ln(b)]$$

   which rearranges to

   $$\ln y = [\ln(b)]x + \ln(a)$$

   in $y = mx + b$ format.

7. To obtain a linear graph, plot $\ln(y)$ versus x.
8. The graph should be linear.
9. Answers will vary. The sample data yields

   $$y = -0.018x + 5.2$$

10. $t_{½} = \dfrac{\ln(2)}{0.018} = 38.5 \text{ s}$

    The half-life found in Question 4 was 37 s, so this result is very close.

Red Hot Half-Life
Modeling Nuclear Decay

Some atoms have unstable nuclei. They will undergo radioactive decay to become more stable. The amount of time it takes for a sample to decay is specific to the type of atom that is decaying. The amount of time it takes for one half of a radioactive sample to decay is called its half-life.

PURPOSE
In this activity you will model radioactive decay with cinnamon candies. The analysis of data and the determination of half-life will be done by graphical means.

MATERIALS

Each lab group will need the following:

beans, navy
calculator, TI® graphing
cinnamon candies
2 cups, 9-oz clear plastic
paper towels
plate, paper

PROCEDURE
1. Count the candies in your cup. Record the number in the data table on your student answer page. This is the value for 0.0 seconds.

2. Put the candies in the cup and then pour them out onto the paper plate. Remove the candies that landed in the starred section. These candies will be considered decayed. Replace the decayed candies with the same number of navy beans so that the total number of particles remains constant. Count the remaining candies and record the number in your data table. Each trial is to be counted as 10 seconds.

3. Continue this procedure until you have ten trials or until you have fewer than 5 cinnamon candies left, whichever comes first. Record each trial in the data table.

4. Answer the questions in the analysis section.

Red Hot Half-Life
Modeling Nuclear Decay

DATA AND OBSERVATIONS

Time (s)	Number of Candies that Decayed	Number of Candies Remaining
0		
10		
20		
30		
40		
50		
60		
70		
80		
90		
100		

ANALYSIS

1. Enter the "time" in L1 and the candies remaining in L2. Be sure that your Stat Plot is set up correctly to display a scatterplot of L1 and L2 data. Describe the shape of the graph.

2. Look at graphs from at least two other groups. What is the same about the graphs? What is different about the graphs?

3. Press STAT ▸ to Calc. Arrow down to **0: ExpReg**. Press ENTER to paste the function; be sure that **Xlist** is L1 and **Ylist** is L2. Arrow down to store **RegEQ**, then VARS to get the variables menu; ▸ to YVARS; ENTER to select 1: Function; ENTER to select Y1: ENTER to execute the command sequence. Press ENTER one last time to calculate.
   Look at the graph by going to ZOOM 9. Write the equation for the line below. Since this is not a linear equation, it will *not* be in the form of $y = mx + b$.

4. Half-life is defined as the amount of time it takes for ½ of a sample to decay. Open Y= and arrow down to Y2. Enter the original number of candies ÷ 2. This corresponds to the time it takes for half of the original sample to decay. Go back to ZOOM 9. Next, go to 2nd TRACE ▾ to 5: Intersection; ENTER to confirm first line; ENTER to confirm second line; ENTER to guess and get the intersection of these two lines. It will be shown at the bottom of your calculator screen. What is the half-life of this system?

5. Take the number of candies that you had at the half-life point. Divide that number by 2. Open Y= and arrow down to Y2. Clear the number and enter the new number. Repeat the procedure that you did in question 4 to determine the intersection of the two lines. Is the new time close to two times the half-life (2 half-lives)?

6. Refer back to question 3. The equation is not linear but instead is in the form of $y = (a)(b)^x$. This is an exponential function. In science we like to have equations in the form of $y = mx + b$. Manipulating the equation, we can do this. It is called a linear transform. In this case, we would take the natural log (ln) of both sides of the equation. Do this with the equation that you have found in question 3. Your teacher will assist you if necessary. Write the equation below.

7. What variables will be used from the equation above to obtain a linear graph?

8. Go back to the statistical function, [STAT] [ENTER] to Edit. You want to transform the data in L2. Arrow up so the cursor is sitting on the L2 icon. Press [LN] [2nd] [2] [ENTER]. Go to [ZOOM] [9] to look at the graph. Is it linear?

9. Go to [Y=] and clear it. Go to [STAT] [▸] to Calc. Arrow down to **4: LinReg**. [ENTER] to select; [VARS] to get to the variables menu; [▸] to YVARS; [ENTER] to select 1: Function; [ENTER] to select Y1; [ENTER] to execute the command sequence. Look at the graph by going to [ZOOM] [9]. Write the equation for the line below.

10. For this type of equation, the absolute value of the slope of the line is equal to a constant that we will refer to as *k*. The half-life is found by the following equation: $t_{1/2} = \frac{\ln(2)}{k}$. Divide ln(2) by the absolute value of your slope. Is it close to the half-life that you found in question 4? Hint: Do not forget to put ( ) around the 2.