Quarknet Teachers Workshop

Summer 2015

Bumble Ball and Random walks

SOURCES:

1. "Modeling Energy Outflow in Stars" by Michael Zeilik
   The Physics Teacher ,Vol. 37, April 1999, pages 236-237

2. "Bumble Rumble: The Journey of a Photon"
   Adler Planetarium, Chicago, Illinois
   Activity Notes 2001-2002

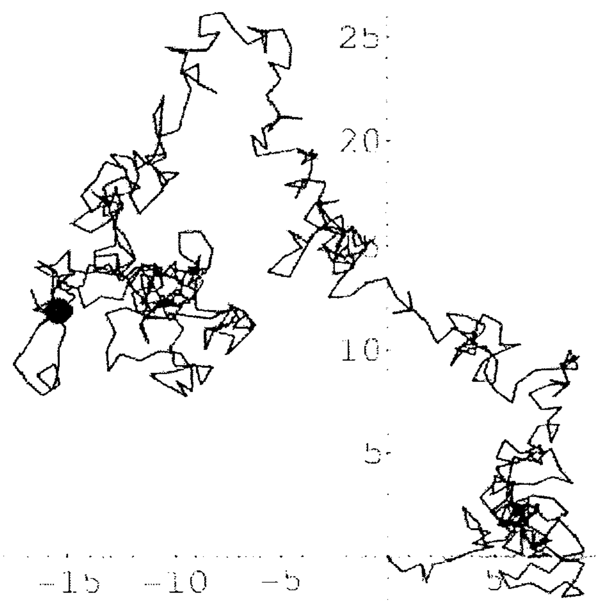
Discussion Questions:

- How can you "see" a black hole?
- How can you "see" atoms and molecules?

Brownian Motion: An example of a "Random Walk"

- Observed by Robert Brown in 1827 (pollen grains floating in water)
- Studied again in 1865 (particles suspended in a sealed liquid kept moving for a whole year)
- Described mathematically by Einstein in 1905 (statistical theory)

A Random Walk in Two Dimensions



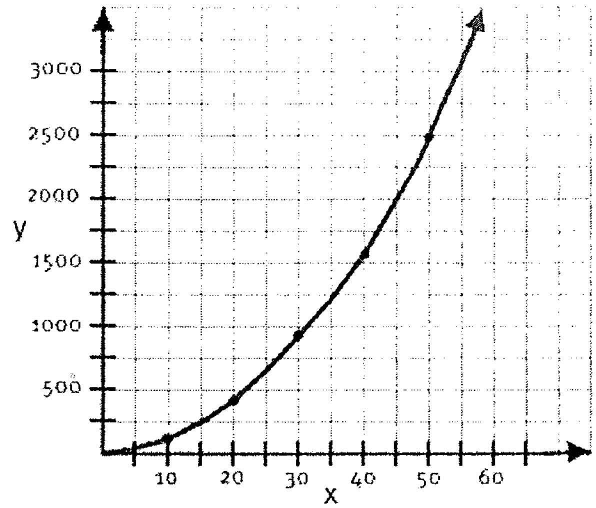
Let r_{average} be the average distance (over many trials) from the walker back to the starting point after a time t has elapsed. How does this average distance increase with time?

$$t = C(r_{average})^2 \quad (C = \text{constant})$$

Graph of y versus x^2 ($y \leftrightarrow t$, $x \leftrightarrow r_{\text{average}}$)

Because this rule has a number squared, its graph will look like a curved line (a "parabola").

The line gets steeper as x increases. That means y is going up, but at a faster rate. It's going up so fast that we don't even have room to show you the rest of the line!



Mathematical Description of Random Walks

$$t = C(r_{average})^2$$

$$C = \frac{(\Delta t)}{n(\Delta r)^2}$$

Δr = step size, Δt = time between steps, n = dimension = 1, 2, 3 .

Example: Photons Produced at the Center of the Sun

$\Delta r = 0.0005$ meters

$\Delta t = \Delta r \div$ (speed of light) = (0.0005÷300,000,000) seconds

$n = 3$

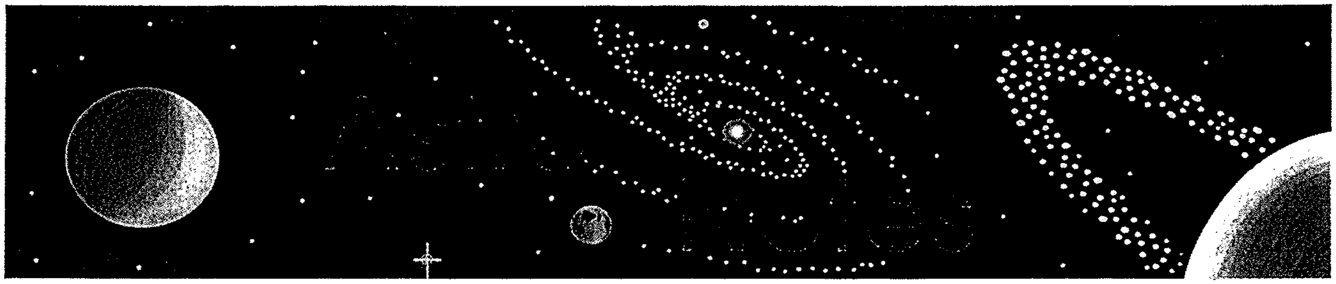
r_{average} = radius of the Sun = 700,000,000 meters

$$\Longrightarrow t \approx 100{,}000 \text{ years}$$

Biased Random Walks

$$t = A(r_{average})^B$$

- $B = 1 \implies$ Constant Velocity (not a random walk)
- $B = 2 \implies$ Random Walk (step direction is truly random)
- $1 < B < 2 \implies$ "Persistent" random walk (still a random walk, but with a directional bias – the direction of motion tends to persist)
- $B > 2 \implies$ "Antipersistent" random walk (the opposite directional bias – the direction of motion tends *not* to persist)



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Modeling Energy Outflow in Stars

Michael Zeilik, The University of New Mexico, Department of Physics and Astronomy, 800 Yale Blvd. N.E., Albuquerque, NM 87131-1156; zeilik@chicoma.la.unm.edu

Do your students have difficulty visualizing how photons travel out of the Sun? Mine do! The most common misunderstanding is that photons formed at the center zip straight out to the surface. That's true for neutrinos, but not for photons.

Fusion reactions generate energy deep within the cores of ordinary stars. How does this energy, originated in part as gamma rays, get to the surface? How much time does it take?

For stars like our Sun, the energy flows outward by two processes: radiative diffusion and convection. The inner 70% of the Sun uses radiation as the primary energy transport system from the core outward. (The outer 30% of the Sun transfers energy by convection.) To help my students develop a physical model of the dominant energy transport in the Sun, we assume that the primary energy transport process is radiative diffusion throughout.

High-energy photons produced in a star's core actually travel incredibly short distances before they are re-absorbed by the gas. This distance is called the photon mean free path and is usually less than 1 mm. Upon absorption, a new photon is immediately re-emitted in *any* direction—possibly even back toward the core! By this process, photons scatter about and slowly progress outward to the surface. Such a process characterizes a *random walk*. This is conceptualized by an object moving in discrete steps, the direction of which is random. Over a large number of steps, an object will make progress away from its starting point. But over a short run of steps (or time), the progress may net zero because of large, random fluctuations.

Over 20 years, I have tried different ways to help students conceptualize a random walk, including computer simulations. But the method I found works the best with a wide range of students is a physical one that uses a clever toy: a Bumble Ball™. It is a plastic sphere about 11 cm in diameter with a symmetrical pattern of rubber knobs extending about 2.5 cm from its surface. The ingenious component is an internal motor that rotates an off-axis mass, which includes the batteries! When activated and placed on a hard surface, a Bumble Ball™ will simulate a random walk. It's a lot of fun for students (and me!) to watch a random walk made so physical (and that's the entertaining point of the toy for infants and toddlers). Although its production by the Ertl Company has been officially discontinued, I have had no problem in finding Bumble Balls™ at local toy stores and discount retailers for about $10.

Activity

As a suggested procedure, first ask your students to make a prediction:

Jeremy Zeilik, age 32 months, playing with a Bumble Ball.™

How long does it take for a photon produced in the Sun's core to reach its surface?

This question will naturally lead to the appropriate physics and the concept of a random walk. Students can then model this concept by determining the mean length of time for a Bumble Ball™ to move a particular distance.

Give each team a Bumble Ball™. Ask them to draw circles of different radii on the floor using easy-to-erase chalk and about 3 m of string. Start with a stopwatch, a circle with a radius of 0.5 m, and five larger circles with increments of 0.5 m.

Each group has the task to measure the time it takes for the ball to move from the center to the edge of one circle. An effective strategy is to have the students arrange themselves around the perimeter to judge when the Bumble Ball™ has exited the circle; this can be a tricky call! The groups should collect the results in one data table (say on the blackboard). This activity requires intergroup cooperation to pull it off successfully over one class period.

The groups *must* do each circle multiple times; otherwise random fluctuations will throw off the results! Five to 10 observations per circle for the larger ones and a greater number for the smaller circles, 30 to 40, will be enough. Each group should calculate a mean time (and standard deviation, if the class is at this level) for their circle.

This is an excellent activity to carry out in cooperative learning teams of about four people. In one class period no one group can carry out the experiment, but it will work if you have about five teams to divide up the task. You will probably need to move out of the classroom and into the halls to find enough space. The surfaces must all be the same and be flat. This activity is fairly noisy; set your class up so it won't disturb others.

The specific analysis of the data depends on the level and the goals of your class. At the very least, you should have the class plot mean elapsed time versus radius of circle. An "eyeball" fit to the data will show that the relation between time and distance is not linear; the curve rises faster than a straight line. If the class is advanced enough, you can ask your students to calculate a "best fit" curve to the data. The crucial information you want from the fit is how the elapsed time varies with the radii of the circles. You expect the function to be some power of the radius; the theoretical value is 2. The slope of a log-log plot is a visual way to find this value; if enough data were taken to minimize random fluctuations, your students will get a value close to 2.

Now note that the "mean elapsed time" is actually a proxy for the number of steps taken by the Bumble Ball™ to travel out of the circle of a given radius. Hence, the number of steps is *proportional to the radius squared.* You need this result to estimate the time it takes for photons to diffuse out of the Sun. For photons, the average step size is roughly 0.5 mm. Next, you need to know the total number of steps for a photon to leave the Sun. That's proportional to the *square* of the Sun's radius (about 700,000 km) divided by the mean step size (0.5 mm), or about 10^{24}. The total distance traveled is 0.5 mm times this number; divide by the speed of light to find the elapsed time. Your result should be about 100,000 years (modern computer models give about 250,000 years due to varying density with depth).

Comment

This activity effectively engages my students. It gives them a very good "feel" for how the fluctuations from the "average" step size really make a Bumble Ball™ travel away from the center of the circle. My students cheer on the ball's progress and sometimes watch it scoot out of the circle in a few bounces, linger right at the edge, or weave back toward the center! A computer simulation just doesn't cut it in comparison.

Note

The Sun and stars are three-dimensional objects, but that does not change the fundamental relationship of the number of steps directly proportional to the radius squared. What it does change is the *constant* of proportionality. For one dimension, it is unity; for two dimensions, it is 2; and for three dimensions, it is 3. I have ignored this factor of three so as to focus on the physics.

Title: Bumble Rumble: The Journey of a Photon

Adapted from *Modeling Energy Outflow in Stars* by Michael Zeilik

Purpose:

To model energy outflow in stars by observing and timing how a bumble ball travels from the center of a circle to outside that circle, which represents the edge of the Radiative Zone.

Objectives:

- Students will be able to identify the Sun's layers and some basic processes that occur in them (fusion, energy transfer, etc.)
- Students will be able to describe photons as packets of light that are produced in the Sun's core.

Intended Audience:

Time Required:

Variable, 60-180 minutes

Materials:

1 Bumble Ball (Can be done with one but may be easier with a few)
1 Data Table per student (found at the end of this lesson)
Sidewalk chalk or masking tape
String
Stopwatch
Clear area at least 6 meters in diameter
Transparency showing the Sun's layers
Overhead projector

Background Information:

In the core, or the center, of the Sun a process called nuclear fusion occurs. Nuclear fusion is the bringing together of atoms to form heavier atoms, resulting in a release of energy. This energy is released in the form of photons. Photons can best be described as packets of light, or clumps of energy. The energy produced in the Sun's core is carried outward by photons. Photons of all energy levels of the Electromagnetic Spectrum are released from the Sun's surface. For example, our eyes use visible light photons to see. Originally, the energy is created as gamma ray photons in the Sun's core and then later transformed to photons of other energy levels along their journey from the center of the Sun's core. Photons travel at the speed of light (186, 000 mi/sec). Photons move very fast, but in the dense core of the Sun their progress is impeded by constant collisions with other photons, atoms, etc., which cause them to move in random directions. Thus, photons take a long time to exit the Sun's core. Photons start out with very short wavelengths, as gamma rays, and lose energy from these collisions. The photons share their energy with the atoms they collide with; these collisions are what keep the Sun hot. The random motion of photons in the Sun's core can be simulated by a jiggling Bumble Ball. Every time the Bumble Ball hits the floor and changes direction, imagine that it's a photon hitting a particle in the interior of the Sun. (Because this is a 2D demo, the floor has to play this role).

After the core and the Radiative Zone, which make up the inner 70 percent of the Sun, the next layer of the sun is the Convective Zone, which is the final 30 percent of the Sun's radius. The Convective Zone is dominated by currents that carry the energy outward to the surface. These convection currents are rising movements of hot gas next to falling movements of cool gas. The convection currents carry photons outward to the surface faster than the radiative transfer that occurs in the core and Radiative Zone.

Once photons reach the surface of the Sun, they are free to head out into space, where there are many fewer particles with which to collide. We are constantly being bombarded by photons (trillions every second), some of which hit our eyes, causing an electrical signal in our brains that we recognize as light.

Preparation:

- Depending upon how much time and space you have, you may either break up students into groups and have each conduct this experiment with a circle of a different radius, or you may want to do this in one big group.
- Whether all of these trials for this demonstration are conducted as one group, or broken up among smaller groups, each of the following trials should be performed:
  - 20-30 trials with a .5 meter radius circle,
  - 20-30 trials with a 1 meter radius circle,
  - 20-30 trials with a 1.5 meter radius circle,
  - 20 or so trials with a 2 meter radius circle, and
  - 20 trials of a 2.5 meter radius circle (if you have time and space).
- For each circle:
  - Measure a piece of string to the length of the radius of the circle.
  - Use easily erased chalk to draw a circle of that radius on the floor. (This works very well on carpet and probably most linoleum flooring, it also comes up easily with a vacuum or wet cloth.)
  - Hold one end of the string firmly to the ground.
  - Pull the string taught and hold the chalk at the other end of the string.
  - Using the first end of the string as the pivot point, and the other end of the string as the guide for the chalk, draw a circle with the chalk on the ground. You may also use tape or string to mark the edge of the circle. To draw circles of larger radii, this may be more easily done with two people, one to hold the string at the center and one to draw the circle.

Procedure:

1. Explain the basics of how the sun works, and what a photon is (see background section).
2. Explain that the Bumble Ball represents a photon.
3. Ask students to make a prediction about how long it takes for a photon produced in the Sun's core to reach its surface. Ask students why they think this might take the amount of time that they predicted.
4. Tell the students that they are going to simulate photons traveling from the center of the core to the beginning of the Convective Zone of the Sun, and discover the relationship between the size of the core and the time it takes photons to exit.

5. Tell the students that the circle represents the core and Radiative Zone of the Sun. Show these to the students on the transparency of the Sun.
6. Place a Bumble Ball in the center of the circle.
7. Coordinate the Bumble Ball's release with the starting of a stopwatch.
8. Time the bumble ball as it travels until it exits the circle.
9. Have students record the results on the Data Table and begin again. There should be high variation in times, especially in the smaller circles, so it is important to do many trials.
10. Have students make observations about the movement of the Bumble Ball (Is there a pattern? Does it seem random? How does bouncing on the floor affect the movement of the ball?). You may want to discuss with older students whether this motion is truly random at all.
11. Have students use the data that they collected on their Data Tables to calculate averages for the times to exit from each diameter circle.
12. Depending upon the level of your students, have them create graphs of the diameter versus the average time to exit. This is most easily done on a computer graphing program like the Chart option in AppleWorks or Excel.
13. For very advanced middle school students or high school students, you may want to have them guess or deduce the relationship between the radius of the "core" and the exit time. At the very least, they should note that the relationship is not linear (meaning that the average time to exit the circle increases proportionally with the increased radius.) The bigger the core, the longer it takes the photon to exit.

Closure:

1. Refer to the Sun transparency and discuss the layers of the Sun with the students.
2. Ask the students what they think will happen to the path and progress of the photons as they travel to less dense layers of the Sun.
3. Tell the students (or have them guess or calculate) that scientists believe that it takes hundreds of thousands of years for a photon to travel from the core to the surface (a distance of 418,000 miles, which is equal to about 50 Earths lined up side by side).

For teacher information only: According to theory, the exit time should be roughly proportional to the radius squared. **The point of this is simply to note that even though it took a very long time for the bumble ball to exit the circle, photons (traveling at the speed of light) still take much, much longer since the radius of the core of the Sun is so great when compared with this circle on the ground.**

Data Table for Bumble Rumble: The Journey of a Photon

Directions: In each cell, record the amount of time in seconds that it takes the Bumble Ball to exit the circle.

Trial Number	0.5 meter radius circle	1.0 meter radius circle	1.5 meter radius circle	2.0 meter radius circle	2.5 meter radius circle
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Calculate average exit times for each circle:

Sum of Column 1: ____________ / ____________ (# of trials) = _______________.
Sum of Column 2: ____________ / ____________ (# of trials) = _______________.
Sum of Column 3: ____________ / ____________ (# of trials) = _______________.
Sum of Column 4: ____________ / ____________ (# of trials) = _______________.
Sum of Column 5: ____________ / ____________ (# of trials) = _______________.

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Objectives:
- Students will be able to identify the Sun's layers and some basic processes that occur in them (fusion, energy transfer, etc.)
- Students will be able to describe photons as packets of light that are produced in the Sun's core.

Intended Audience:
8-12th grades – From basic inquiry of photon movement to mathematical equations and graphing.

Time Required:
Variable, 60-180 minutes

Materials:
1 Bumble Ball (Can be done with one but may be easier with a few)
1 Data Table per student (found at the end of this lesson)
Sidewalk chalk or masking tape
String
Stopwatch
Clear area at least 6 meters in diameter
Transparency showing the Sun's layers
Overhead projector

Background Information:
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Data Table for Bumble Rumble: The Journey of a Photon

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Trial Number	0.5 meter radius circle	1.0 meter radius circle	1.5 meter radius circle	2.0 meter radius circle	2.5 meter radius circle
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Calculate average exit times for each circle:

Sum of Column 1: ____________ / ____________ (# of trials) = _______________.
Sum of Column 2: ____________ / ____________ (# of trials) = _______________.
Sum of Column 3: ____________ / ____________ (# of trials) = _______________.
Sum of Column 4: ____________ / ____________ (# of trials) = _______________.
Sum of Column 5: ____________ / ____________ (# of trials) = _______________.

Data Table for Bumble Rumble: The Journey of a Photon

Directions: In each cell, record the amount of time in seconds that it takes the Bumble Ball to exit the circle.

Trial Number	0.5 meter radius circle	1.0 meter radius circle	1.5 meter radius circle	2.0 meter radius circle	2.5 meter radius circle
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Calculate average exit times for each circle:

Sum of Column 1: ____________ / ____________ (# of trials) = _______________.
Sum of Column 2: ____________ / ____________ (# of trials) = _______________.
Sum of Column 3: ____________ / ____________ (# of trials) = _______________.
Sum of Column 4: ____________ / ____________ (# of trials) = _______________.
Sum of Column 5: ____________ / ____________ (# of trials) = _______________.

$$t = 53.58\ r^{1.278}$$

(Very) Persistent Random Walk

	Data Set	
	Radius (meters)	Time (seconds)
1	0.5	16.76
2	1	46.6
3	1.5	98.2
4	2	135.24
5	2.5	167.36
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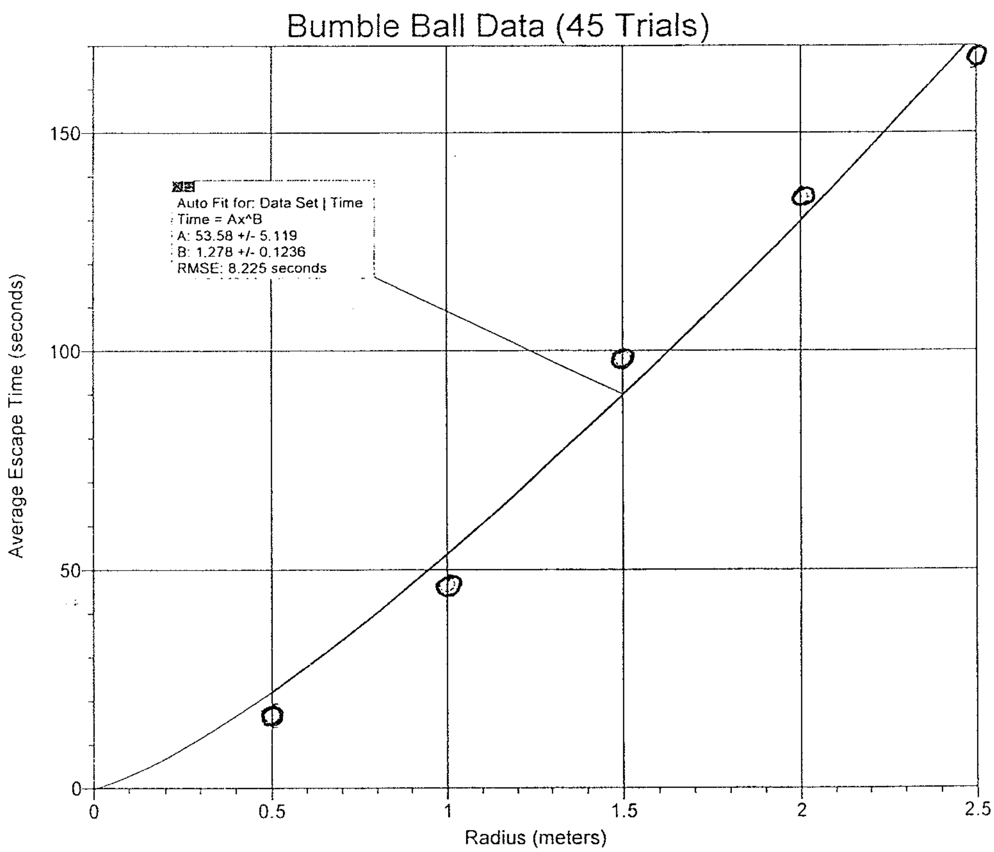

Data Table for Bumble Rumble: The Journey of a Photon

Directions: In each cell, record the amount of time in seconds that it takes the Bumble Ball to exit the circle.

Trial Number	0.5 meter radius circle	1.0 meter radius circle	1.5 meter radius circle	2.0 meter radius circle	2.5 meter radius circle
1	36	195	223	330	402
2	22	148	200	216	233
3	8	22	29	40	82
4	30	46	108	113	114
5	10	35	40	110	125
6	16	29	60	66	81
7	4	21	40	51	61
8	5	9	22	28	32
9	59	126	150	180	193
10	50	74	122	133	321
11	22	36	48	63	150
12	48	59	94	110	152
13	47	67	93	103	126
14	11	28	51	100	102
15	9	66	108	125	138
16	5	20	50	89	126
17	17	37	65	75	120
18	3	24	155	191	196
19	12	30	39	129	146
20	6	33	47	66	96
21	9	23	161	193	211
22	3	13	67	163	175
23	11	23	28	36	46
24	10	61	75	89	137
25	29	111	201	396	410
26	2	108	120	145	153
27	3	57	114	126	258
28	4	16	114	124	130
29	5	19	223	230	304
30	10	12	41	112	118

Calculate average exit times for each circle:

Sum of Column 1: ____________ / ____________ (# of trials) = ______________.
Sum of Column 2: ____________ / ____________ (# of trials) = ______________.
Sum of Column 3: ____________ / ____________ (# of trials) = ______________.
Sum of Column 4: ____________ / ____________ (# of trials) = ______________.
Sum of Column 5: ____________ / ____________ (# of trials) = ______________.

Data Table for Bumble Rumble: The Journey of a Photon

Directions: In each cell, record the amount of time in seconds that it takes the Bumble Ball to exit the circle.

Trial Number	0.5 meter radius circle	1.0 meter radius circle	1.5 meter radius circle	2.0 meter radius circle	2.5 meter radius circle
1	3	8	34	126	147
2	17	28	74	108	118
3	15	29	210	234	245
4	17	51	80	97	163
5	50	113	246	307	381
6	5	10	92	182	205
7	11	21	65	101	174
8	9	21	28	81	91
9	21	26	41	59	100
10	14	30	53	67	77
11	38	51	151	178	198
12	23	43	152	189	210
13	6	29	62	156	165
14	14	29	127	134	158
15	5	60	116	135	161
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Calculate average exit times for each circle:

Sum of Column 1: 754 / 45 (# of trials) = 16.76.
Sum of Column 2: 2097 / 45 (# of trials) = 46.60.
Sum of Column 3: 4419 / 45 (# of trials) = 98.20.
Sum of Column 4: 6086 / 45 (# of trials) = 135.24.
Sum of Column 5: 7531 / 45 (# of trials) = 167.36.

$$t = 43.66\, r^{1.220}$$

(Very) Persistent Random Walk

	Data Set	
	Radius (meters)	t (seconds)
1	0.5	13.42
2	1.0	45.62
3	1.5	74.09
4	2.0	101.34
5	2.5	132.64
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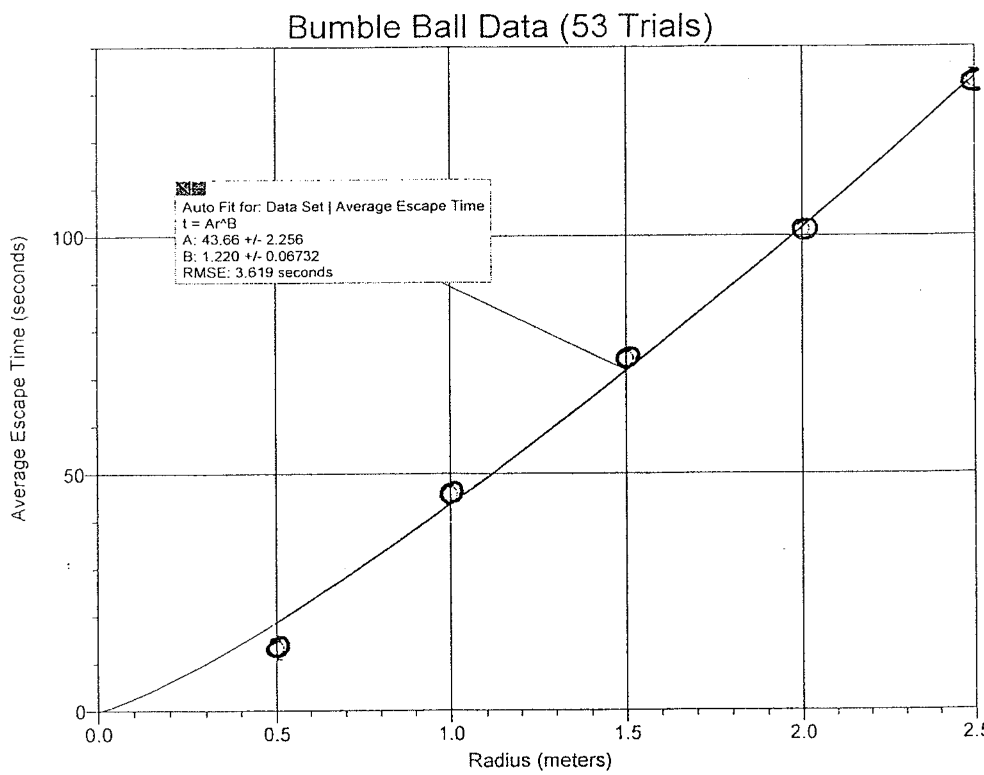

Group B

Data Table for Bumble Rumble: The Journey of a Photon

Directions: In each cell, record the amount of time in seconds that it takes the Bumble Ball to exit the circle.

Trial Number	0.5 meter radius circle	1.0 meter radius circle	1.5 meter radius circle	2.0 meter radius circle	2.5 meter radius circle
1	2	4	7	15	20
2	11	27	57	193	206
3	7	52	68	84	110
4	10	27	85	105	195
5	7	18	23	27	39
6	16	160	183	189	192
7	16	71	86	95	102
8	14	111	212	345	454
9	2	107	310	315	328
10	15	330	438	512	536
11	8	20	26	93	99
12	4	30	46	67	101
13	4	23	27	30	36
14	2	35	38	74	75
15	4	8	10	18	48
16	7	15	21	57	111
17	2	6	21	101	136
18	5	16	27	34	55
19	17	30	31	77	117
20	3	21	140	150	163
21	17	27	40	97	117
22	6	65	70	84	118
23	7	25	31	35	73
24	18	31	40	49	53
25	10	19	47	62	71
26	5	12	16	27	44
27	11	24	29	62	99
28	10	22	31	37	60
29	7	14	58	66	78
30	17	44	62	77	84

Calculate average exit times for each circle:

Sum of Column 1: 264 / 30 (# of trials) = 8.80.
Sum of Column 2: 1394 / 30 (# of trials) = 46.47.
Sum of Column 3: 2280 / 30 (# of trials) = 76.00.
Sum of Column 4: 3177 / 30 (# of trials) = 105.90.
Sum of Column 5: 3920 / 30 (# of trials) = 130.67.

Group A

Data Table for Bumble Rumble: The Journey of a Photon

Directions: In each cell, record the amount of time in seconds that it takes the Bumble Ball to exit the circle.

Trial Number	0.5 meter radius circle	1.0 meter radius circle	1.5 meter radius circle	2.0 meter radius circle	2.5 meter radius circle
1	4	10	17	27	45
2	6	16	40	116	243
3	32	52	101	127	130
4	5	19	57	106	117
5	12	20	38	52	126
6	138	154	179	202	229
7	7	15	30	70	101
8	20	76	86	110	121
9	8	21	34	58	89
10	6	11	77	93	101
11	14	17	99	123	135
12	64	69	78	110	124
13	8	20	55	76	119
14	3	22	63	69	218
15	12	56	122	129	138
16	23	115	146	154	163
17	22	82	90	109	147
18	5	32	40	46	86
19	26	~~105~~ 105	116	136	186
20	2	32	63	84	104
21	2	16	25	69	108
22	12	28	~~[illegible]~~ 38	58	191
23	16	36	53	70	89
24					
25					
26					
27					
28					
29					
30					

Calculate average exit times for each circle:

Sum of Column 1: 447 / 23 (# of trials) = 19.43.
Sum of Column 2: 1024 / 23 (# of trials) = 44.52.
Sum of Column 3: 1647 / 23 (# of trials) = 71.61.
Sum of Column 4: 2194 / 23 (# of trials) = 95.39.
Sum of Column 5: 3110 / 23 (# of trials) = 135.22.