S-09

E&M

January 2009

1 Capacitors

Consider a spherical capacitor which has the space between its plates filled with a dielectric of permittivity ϵ. The inner sphere has radius r_1 and the outer sphere has radius r_2. The total free charge on the capacitor is Q.

a) Find the electric displacement $\vec{\mathbf{D}}$, and the electric field $\vec{\mathbf{E}}$ at a radius r inside the dielectric. (**2.5-points**)

b) Find the electric energy density u_e inside the dielectric. (**2.5-points**)

c) Find the total energy U_e of the capacitor. (**2.5-points**)

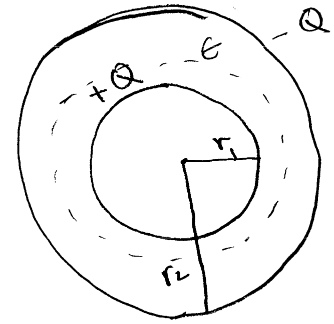
d) Find the capacitance C, of the capacitor. (**2.5-points**)

1.

(Gaussian)

a)

Using Gauss's law at $r_1 < r < r_2$



$$\int E \cdot d\bar{a} = 4\pi Q$$

$$\Rightarrow \quad E \cdot 4\pi r^2 = 4\pi Q$$

$$\Rightarrow \quad \bar{E} = \frac{Q}{r^2}\hat{r}$$

$$\therefore \quad \bar{D} = \epsilon \bar{E} = \frac{\epsilon Q}{r^2}\hat{r}$$

b) For a dielectric

$$u_e = \frac{1}{8\pi}(\bar{H}\cdot\bar{B} + \bar{E}\cdot\bar{D})$$

There is no B-field Thus, $u_e = \frac{1}{8\pi}(\bar{E}\cdot\bar{D})$

$$\Rightarrow u_e = \frac{1}{8\pi}\epsilon\frac{Q^2}{r^4}$$

c) total energy $U_e = \int u_e \, d^3x = \frac{\epsilon Q^2}{8\pi}\int \frac{1}{r^4} r^2 dr \sin\theta d\theta d\phi$

$$= 4\pi \frac{\epsilon Q^2}{8\pi}\int_{r_1}^{r_2} \frac{1}{r^2} dr$$

$$= \frac{\epsilon Q^2}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

d)

capacitance, $C = \frac{Q}{V}$

Inside the capacitor $\bar{E} = \frac{Q}{r^2}\hat{r}$

$$\Rightarrow V = -\int \bar{E}\cdot d\bar{r}$$

$$= -\int_{r_1}^{r_2} \frac{Q}{r^2}\,dr$$

$$= -Q\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\therefore \quad C = \left(\frac{1}{r_2} - \frac{1}{r_1}\right)^{-1}$$

2 Rods

In this problem you will determine the magnetic field produced by three different infinitely long, cylindrical conducting rods. The figures on the next page are useful for vizualizing the differences in the rods.

a) State whether you are using MKS or cgs units (**0-points**).

b) **Rod 1**: Consider an infinitely long, solid, cylindrical conducting rod (known as Rod 1) with radius $2a$ that is concentric with the z-axis and carries a uniform current density $+J_0$ in the $+z$ direction (Fig. 1). Let $r = (x^2+y^2)^{1/2}$ be the perpendicular distance to the z-axis and θ be the angle r makes with the positive x-axis. (See Fig. 1.) Find the magnitude and direction of the magnetic field $\vec{\mathbf{B}}_1$ produced by Rod 1 for all r. Give the direction in Cartesian coordinates using the unit vectors: $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$. (**3-points**)

c) **Rod 2**: Consider a second, infinitely long, solid, cylindrical conducting rod (i.e. Rod 2) that is parallel to the z-axis and has radius a. The axis of Rod 2 is centered at $(x, y) = (+a, 0)$. Rod 2 carries a uniform current density $-J_0$ in the $-z$ direction. Let ρ be the radial distance from the axis of the Rod 2 and let ϕ be the angle that ρ makes with the positive x-axis. (See Fig. 2.) Find the magnitude and direction of the magnetic field $\vec{\mathbf{B}}_2$ produced by Rod 2 for all values of ρ and ϕ. Give the direction in Cartesian coordinates using the unit vectors: $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$. (**1-points**)

d) **Rod 3**: Consider an infinitely long, cylindrical conducting rod (i.e. Rod 3) with radius $2a$ that is concentric with the z-axis and carries a uniform current density J_0 in the $+z$ direction. However in this conductor, an (infinitely long) hole of radius a is drilled parallel to the z-axis at the position $(x, y) = (+a, 0)$. (See Fig 3). Find the magnitude and direction of the magnetic field $\vec{\mathbf{B}}_3$ produced by Rod 3 on the x-axis (at $y = 0$) for all values of $x > 0$. Give the direction in the Cartesian cordinates using the unit vectors: $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$. (**6-points**)

2.

(Gaussian)

$r > 2a$

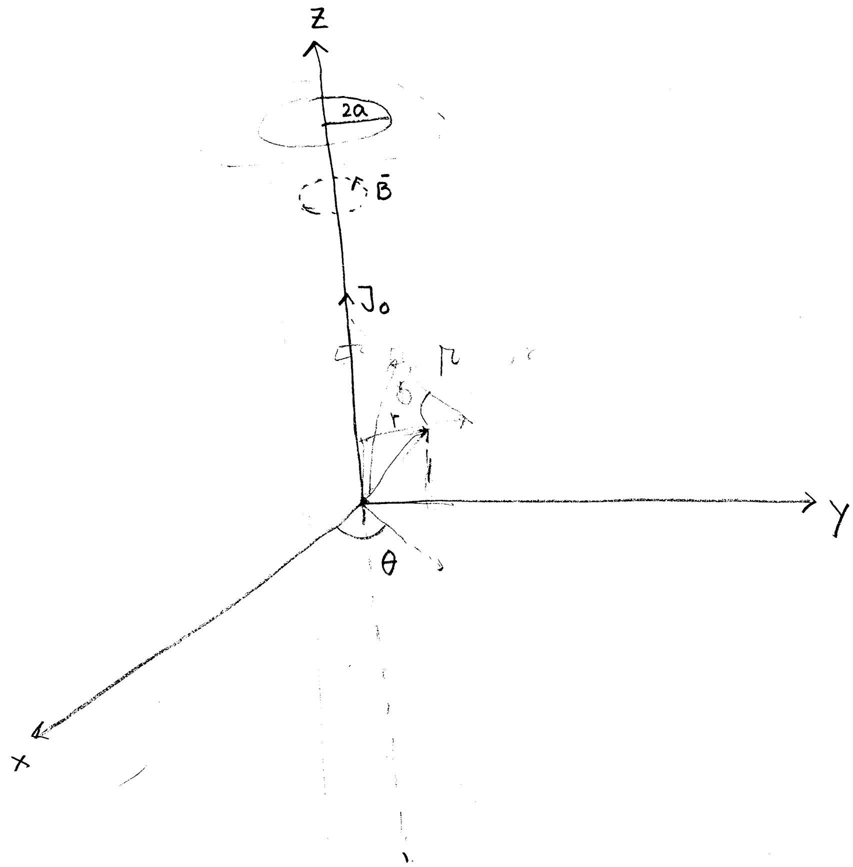
$$\oint \bar{B}\cdot d\bar{l} = \int \bar{J}\cdot d\bar{a}$$

$$\Rightarrow B\cdot 2\pi r = \int J_0\, r\,dr\,d\phi$$

$$= 2\pi J_0 \frac{r^2}{2}\Big|_0^{2a}$$

$$= 4\pi a^2 J_0$$

$$\Rightarrow \bar{B} = \frac{2a^2 J_0}{r}\hat{\phi}$$



Using Biot-Savart

$\bar{r}-\bar{r}'$

$\bar{r}' = z'\hat{k}$

$\bar{r} = x\hat{i} + y\hat{j}$

i) $$\bar{B}(r) = \frac{1}{c}\int \frac{\bar{J}(\bar{r}')\times\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$= \frac{1}{c}\int \underbrace{\bar{J}(x',y',z')}_{J_0\hat{k}} \times \frac{x\hat{i}+y\hat{j}-z'\hat{k}}{(x^2+y^2+z'^2)^{3/2}}$$

$|\bar{r}-\bar{r}'| = \sqrt{r^2+r'^2-2rr'}$

$$= \frac{1}{c}\int \frac{xJ_0\hat{j} - yJ_0\hat{i}}{(x^2+y^2+z'^2)^{3/2}}\,d\tau'$$

$|\bar{r}-\bar{r}'| = \sqrt{x^2+y^2+z'^2}$

$$= \frac{1}{c}\int \frac{J_0\hat{j} - J_0\hat{i}}{(r^2+z'^2)^{3/2}}\,r'dr'd\theta'dz' = \frac{4\pi a^2 J_0}{c}(x\hat{j}-y\hat{i})\int_{-\infty}^{+\infty}\frac{dz'}{(r^2+z'^2)^{3/2}}$$

$$z' = r\cot x$$

$$dz' = -r\csc^2 x\, dx$$

$$\theta = \cot^{-1}\left(\frac{z'}{r}\right)$$

$$z' = +\infty \Rightarrow \theta = 0$$

$$z' = -\infty \Rightarrow \theta = \pi$$

Now,

$$\int_{-\infty}^{+\infty} \frac{dz'}{(r^2+z'^2)^{3/2}}$$

$$= \int_{\pi}^{0} \frac{-r\csc^2\theta\, d\theta}{r^3\csc^3\theta}$$

$$= \int_{0}^{\pi} \frac{1}{r^2}\sin\theta\, d\theta$$

$$= \frac{1}{r^2}\Big[-\cos\theta\Big]_0^{\pi} = \frac{1}{r^2}(1+1) = \frac{2}{r^2}$$

$$\bar{B}(\bar{r}) = \frac{8\pi a^2 J_0}{c r^2}(x\hat{j} - y\hat{i})$$

$$= \frac{8\pi a^2 J_0}{c r}(\cos\theta\,\hat{j} - \sin\theta\,\hat{i})$$

ii) $\bar{r}' = a\hat{i} - z\hat{k}$

$\bar{r} = (a+x)$

3 Cubical Box

A cubical box (sides of length a) consists of five metal plates, which are welded together and grounded. The top is made of a separate sheet of metal, insulated from the others, and held at constant potential V_0. In this problem you will find the potential **inside** the box.

a) Assume that Laplace's Equation is separable and, beginning with Laplace's Equation, write three ordinary, second-order, differential equations, one each for x, y, and z. How are these equations linked? (**2-points**)

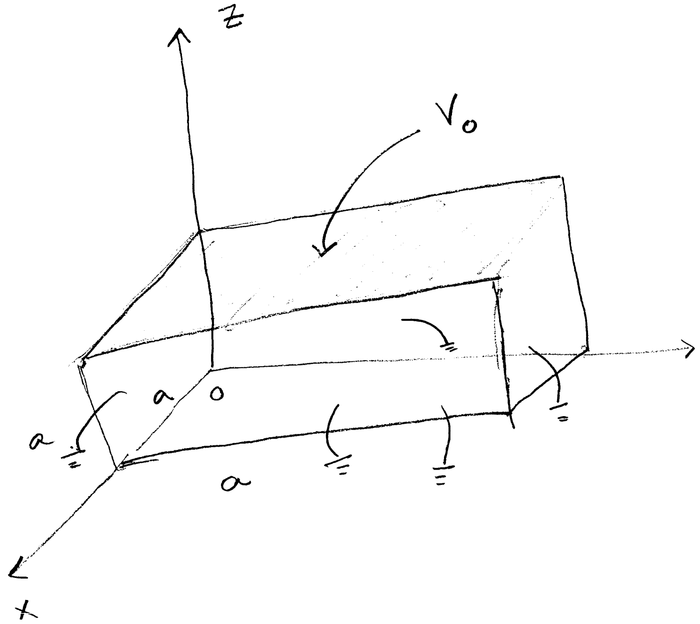
b) Write the appropriate forms of the solutions to the three differential equations of part (a). (**2-points**)

c) Apply the boundary conditions to determine all constants for the solutions of part (b). (**4-points**)

d) From your results in part (c), construct the general solution for the potential $V(x, y, z)$ inside the box. (**2-points**)

3. (Gaussian)

Now charge inside the box ⇒ pot. satisfies Laplace's eqn



$$\nabla^2 V = 0$$

$$\Rightarrow \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

Assuming Laplaces eqn is seperable, we can write the potential as

$$V(x,y,z) = X(x)\,Y(y)\,Z(z)$$

$$\Rightarrow YZ\frac{d^2X}{dx^2} + ZX\frac{d^2Y}{dy^2} + XY\frac{d^2Z}{dz^2} = 0$$

$$\Rightarrow \frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = 0$$

b) Let,

$$\frac{1}{Z}\frac{d^2Z}{dz^2} = +l^2+k^2 \Rightarrow Z(z) = E\,e^{\sqrt{l^2+k^2}\,z} + F e^{-\sqrt{l^2+k^2}\,z}$$

$$\frac{1}{Y}\frac{d^2Y}{dy^2} = -k^2 \Rightarrow Y(y) = C\sin ky + D\cos ky$$

$$\therefore \frac{1}{X}\frac{d^2X}{dx^2} = -l^2 \Rightarrow X(x) = A\sin lx + B\cos lx$$

c) Boundary conditions

i) $x = 0; \quad V = 0$

ii) $x = a; \quad V = 0$

iii) $y = 0; \quad V = 0$

iv) $y = a; \quad V = 0$

v) $z = 0; \quad V = 0$

vi) $z = a; \quad V = V_0$ * BC on z?

vii) $z \to \infty \quad V \to 0$

BC

~ $z = 0 \quad V = 0$

$$0 = E + F \Rightarrow E = -F$$

~ $z = a \quad V = V_0$

$$V_0 = E e^{\sqrt{k^2+l^2}\, a} - E e^{-\sqrt{k^2+l^2}\, a}$$

$$\Rightarrow F \left(e^{\sqrt{k^2+l^2}\, a} - e^{-\sqrt{k^2+l^2}\, a} \right) = V_0$$

$$F = \frac{2V_0}{\sinh(\sqrt{k^2+l^2}\, a)}$$

~ $z \to \infty \quad V \to 0$

$$Z(z) = F e^{-\sqrt{l^2+k^2}\, z}$$

$\rightsquigarrow y = 0, \quad V = 0$

$0 = D$

$\rightsquigarrow y = a, \quad V = 0$

$0 = C \sin ka$

$\Rightarrow \sin n\pi = \sin ka$

$\Rightarrow k = \frac{n\pi}{a}$

$\rightsquigarrow x = 0, \quad V = 0$

$0 = B$

$\rightsquigarrow z = a, \quad V = 0$

$0 = A \sin la$

$\Rightarrow l = \frac{m\pi}{a}$

$$\therefore V(x,y,z) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{nm} \frac{2V_0 \sinh\left(\frac{\sqrt{n^2\pi^2 + m^2\pi^2}}{a} z\right)}{\sinh\left(\sqrt{n^2\pi^2 + m^2\pi^2}\right)} \cos\left(\frac{n\pi}{a}\right) \sin\left(\frac{m\pi}{a}\right)$$

$$V(x,y,z) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{m,n}\left(e^{\frac{\sqrt{n^2\pi^2+m^2\pi^2}}{a}z} - e^{-\frac{\sqrt{n^2\pi^2+m^2\pi^2}}{a}z}\right)$$

$$\sin\left(\frac{n\pi}{a}y\right)\sin\left(\frac{m\pi}{a}x\right)$$

$$\Rightarrow V(x,y,a) = V_0 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}\; 2\sinh\left(\sqrt{n^2+m^2}\,\pi\right)\sin\left(\frac{n\pi}{a}y\right)$$

$$\sin\left(\frac{m\pi}{a}x\right)$$

$$\Rightarrow \int_0^a\int_0^a V_0 \sin\left(\frac{n'\pi}{a}y\right)\sin\left(\frac{m'\pi}{a}\right)x \; dy\,dx$$

$$= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}\, 2\sinh\left(\sqrt{n^2+m^2}\,\pi\right)\int_0^a \sin\left(\frac{n\pi}{a}y\right)\sin\left(\frac{n'\pi}{a}y\right)dy$$

$$\int_0^a \sin\left(\frac{m\pi}{a}z\right)\sin\left(\frac{m'\pi}{a}z\right)dz$$

$$\Rightarrow C_{mn}\, 2\sinh\left(\sqrt{n^2+m^2}\,\pi\right)\frac{a^2}{4} = V_0\int_0^a \sin\left(\frac{n\pi}{a}y\right)dy\int_0^a \sin\left(\frac{m\pi}{a}z\right)dz$$

$$\Rightarrow C_{mn} = \frac{2V_0}{a^2\sinh\left(\sqrt{n^2+m^2}\,\pi\right)}\,\frac{a}{n\pi}(1-\cos n\pi)\,\frac{a}{m\pi}(1-\cos m\pi)$$

$$= \begin{cases} \dfrac{16V_0}{nm\pi^2\sinh\left(\sqrt{n^2+m^2}\right)\pi}; & m,n \text{ odd} \\ 0 & ;\ m,n \text{ even} \end{cases}$$

$$\therefore\ V(x,y,z) = \frac{16V_0}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{nm} \frac{\sinh\left(\frac{\sqrt{n^2+m^2}}{a}\pi\right)z}{\sinh\left(\sqrt{n^2+m^2}\,\pi\right)} \sin\left(\frac{n\pi}{a}y\right) \cos\left(\frac{m\pi}{a}z\right)$$

$m, n \Rightarrow$ odd

4 Conducting Sphere

In this problem you are to prove that a perfectly conducting sphere acquires a magnetic dipole moment when placed in a uniform magnetic field $\vec{\mathbf{B}}_0$. Let the sphere have radius a. By perfectly conducting, we mean that there is no magnetic field in the interior of the sphere. As a result of the induced dipole moment, the magnetic field $\vec{\mathbf{B}}$ exterior to the sphere is no longer $\vec{\mathbf{B}}_0$. Determine the dipole moment as follows.

a) What are the boundary conditions on $\hat{\mathbf{n}} \cdot \vec{\mathbf{B}}$ and $\hat{\mathbf{n}} \times \vec{\mathbf{B}}$ at the surface of the sphere, where $\hat{\mathbf{n}}$ is the outward unit normal to the spherical surface? These boundary conditions involve the surface current density $\vec{\mathbf{K}}$, which will be determined below. (**1-point**)

b) Assume that the induced magnetic field is a pure magnetic dipole field, that is, in a spherical cordinate system with origin at the center of the sphere,

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_0 + \frac{3\vec{\mu} \cdot \vec{\mathbf{r}}\vec{\mathbf{r}} - \vec{\mu} r^2}{r^5}, \qquad r > a.$$

Use the boundary condition on $\hat{\mathbf{r}} \cdot \vec{\mathbf{B}}$ at $r = a_+$ to determine $\vec{\mu}$ in terms of a and $\vec{\mathbf{B}}_0$. (**3-points**)

c) Use the boundary condition on $\hat{\mathbf{r}} \times \vec{\mathbf{B}}$ at $r = a_+$ to determine the surface current density $\vec{\mathbf{K}}$ in terms of $\vec{\mathbf{r}}$ and $\vec{\mathbf{B}}_0$. (**3-points**)

d) Compute the magnetic dipole moment from the surface current according to

$$\vec{\mu} = \frac{1}{2c} \oint dS \, \hat{\mathbf{r}} \times \vec{\mathbf{K}}$$

where the integral extends over the surface of the sphere, and show that $\vec{\mu}$ coincides with the result found in part b. (**3-points**)

Prob 4 (Gaussian)

a) $\underline{\bar{B}.\hat{n}}$

i) $B^{\perp}_{above} - B^{\perp}_{below} = 0$

$\underline{\hat{n} \wedge \bar{B}}$

ii) $H^{\parallel}_{above} - H^{\parallel}_{below} = \frac{4\pi}{c} \bar{K}_f \wedge \hat{n}$

b) $\bar{B} = 0 \qquad r < a$

$$\bar{B} = \bar{B}_0 + \frac{3\bar{\mu}.\bar{r}\bar{r} - \vec{\mu} r^2}{r^5} \quad , \quad r > a$$

$$\hat{r}.\bar{B} = \hat{r}.\bar{B}_0 + \frac{3(\bar{\mu}.\bar{r})\overbrace{(\hat{r}.\bar{r})}^{r} - (\hat{r}.\vec{\mu}) r^2}{r^5}$$

So,

$$\hat{r}.\bar{B}\Big|^{above}_{r=a_+} - \hat{r}.\bar{B}\Big|^{below}_{r=a_+} = 0$$

$$\Rightarrow \quad \hat{r}.\bar{B}_0 + \frac{3a_+^2(\hat{r}.\vec{\mu}) - a_+^2(\hat{r}.\vec{\mu})}{a_+^5} = 0$$

$$\hat{r}.\bar{B}_0 a_+^5 = -2 a_+^2 (\hat{r}.\vec{\mu})$$

$$\Rightarrow \quad \hat{r}.\vec{\mu} = \hat{r}.(\bar{B}_0 a_+^3) \Rightarrow \vec{\mu} = -\frac{1}{2}\bar{B}_0 a_+^3$$

c) $$\bar{B} = \bar{B}_0 + \frac{3\bar{\mu}\cdot\bar{r}\,\bar{r} - \bar{\mu} r^2}{r^5} \qquad r > a$$

$$\hat{r}\wedge\bar{B} = \hat{r}\wedge\bar{B}_0 + \frac{3\bar{\mu}\cdot\bar{r}\,\overbrace{(\hat{r}\wedge\bar{r})}^{=0} - (\hat{r}\wedge\vec{\mu})r^2}{r^5}$$

$$= \hat{r}\wedge\bar{B}_0 - \frac{(\hat{r}\wedge\vec{\mu})}{r^3}$$

Boundary conditions

$$\hat{r}\wedge\bar{H}^{above}\Big|_{r=a_+} - \underbrace{\hat{r}\wedge H^{below}}_{=0}\Big|_{r=a_+} = \frac{4\pi}{c}\bar{K}_f\wedge\hat{r}$$

$$\Rightarrow \quad (\hat{r}\wedge\bar{B})\Big|_{r=a_+} = \frac{4\pi}{c}\bar{K}_f\wedge\hat{r}$$

$$\Rightarrow \quad \hat{r}\wedge\bar{B}_0 - \frac{(\hat{r}\wedge\vec{\mu})}{a_+^3} = \frac{4\pi}{c}K_f\wedge\hat{r}$$

$$\Rightarrow \quad \hat{r}\wedge\bar{B}_0 + \frac{1}{2}(\hat{r}\wedge\bar{B}_0) = \frac{4\pi}{c}\bar{K}_f\wedge\hat{r}$$

$$\Rightarrow \hat{r}\wedge\left(\frac{3}{2}B_0\right) = \hat{r}\wedge\left(-\frac{4\pi}{c}\overleftarrow{K}_f\right)$$

$$\Rightarrow \quad \vec{K} = -\frac{3c}{8\pi}\bar{B}_0$$

d)

$$\vec{\mu} = \frac{1}{2c} \oint ds \ \hat{r} \wedge \vec{K}$$

$$= \frac{1}{2c} \int a_t^2 \sin\theta \, d\theta \, d\phi \ \hat{r} \wedge \left(-\frac{3c}{8\pi} \bar{B}_0 \right)$$

$$= -\frac{3}{4} \ \hat{r} \wedge \bar{B}_0$$

$$=$$

5 Stress Tensor

Consider a stationary solid sphere of radius a and uniform surface charge density σ. Assume a coordinate system for which the sphere is centered at the origin.

a) Specify the system of units you will be using. (**0-points**)

b) Determine the electromagnetic field everywhere on the x-y plane (**2-points**)

c) Write down the Maxwell Stress Tensor everywhere. (**4-points**)

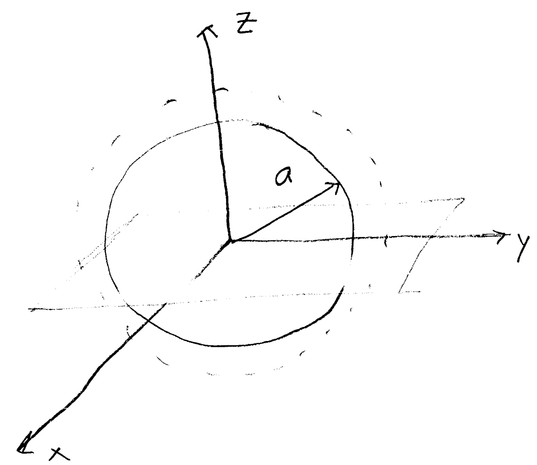
d) Use the Maxwell Stress Tensor to determine the net force that the southern hemisphere ($z < 0$) exerts on the northern hemisphere ($z > 0$). (**4-points**)

Q: $\rho = 0$?

Prob 5 (Gaussian)

a) Assuming no vol charge density $\rho = 0$

and only uniform surface density σ



Using a gaussian surface of radius r

• $r > a$

$$\int \bar{E} \cdot d\bar{a} = 4\pi \int \sigma \, da$$

$$\Rightarrow \int E \, r^2 \, d\theta \, d\phi = 4\pi\sigma \int a^2 \, d\theta \, d\phi$$

$$\Rightarrow E \, (4\pi r^2) = 4\pi\sigma \, (4\pi a^2)$$

$$\Rightarrow \bar{E} = \frac{4\pi\sigma a^2}{r^2} \hat{r}$$

Thus on the x-y plane,

$$\bar{E}(x,y) = \frac{4\pi\sigma a^2}{(x^2+y^2)^{3/2}} (x\hat{x} + y\hat{y}) \quad (r > a)$$

• $r < a$

$$\bar{E}(x,y) = 0$$

c) $$T_{\alpha\beta} = \frac{1}{4\pi}\left(E_\alpha D_\beta + H_\alpha B_\beta - \frac{1}{2}\left(\bar{E}\cdot\bar{D} + \bar{B}\cdot\bar{H}\right)\delta_{\alpha\beta}\right)$$

$$T_{xy} = \frac{1}{4\pi}\left(E_x E_y - \frac{1}{2} E^2 \delta_{xy}\right)$$

$$T^{\alpha\beta} = \frac{1}{4\pi}\begin{pmatrix} \frac{(4\pi)^2\sigma^2 a^4 x^2}{(x^2+y^2)^3} - \frac{(4\pi)^2\sigma^2 a^4}{(x^2+y^2)^2} & \frac{(4\pi\sigma)^2 a^4 xy}{(x^2+y^2)^3} \\ \frac{(4\pi\sigma)^2 a^4 xy}{(x^2+y^2)^3} & \frac{(4\pi\sigma)^2 a^4 y^2}{(x^2+y^2)^3} - \frac{(4\pi\sigma)^2 a^4}{(x^2+y^2)^2} \end{pmatrix}$$

The electric field

6 Electromagnetic Waves

A monochromatic, plane polarized, plane electromagnetic wave traveling in the z-direction in the lab frame (in a vacuum, $\epsilon = \mu = 1$) can be written in the following 3+1 dimensional form:

$$\vec{\mathbf{E}} = E_0 \hat{\mathbf{i}} \exp^{\imath(kz-\omega t)},$$

$$\vec{\mathbf{B}} = B_0 \hat{\mathbf{j}} \exp^{\imath(kz-\omega t)},$$

a) Combine this $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ into a single electromagnetic field tensor $F^{\alpha\beta}$ and use Maxwell's equations in the 4-dimensional form

$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0$$

$$\partial_\alpha F^{\alpha\beta} = \frac{4\pi}{c} J^\beta = 0$$

to find all constraints on the 4 constants E_0, B_0, k, and ω (i.e., the above wave won't satisfy Maxwell's equations for arbitrary values of all four of these parameters). **(2-points)**

b) What are the values of the invariants $F^{\alpha\beta}F_{\alpha\beta}$ and $\epsilon^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}$ for this wave? **(2-points)**

c) Use a Lorentz boost to find $F'^{\alpha\beta}$ in a frame moving in the +z direction with a speed ν. Don't forget to express your answer in terms of the moving coordinates t' and x'^i. **(2-points)**

d) How has the frequency and the wavelength of this wave changed in the moving frame? **(2-points)**

e) How has $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ changed in direction and/or magnitude? **(2-points)**