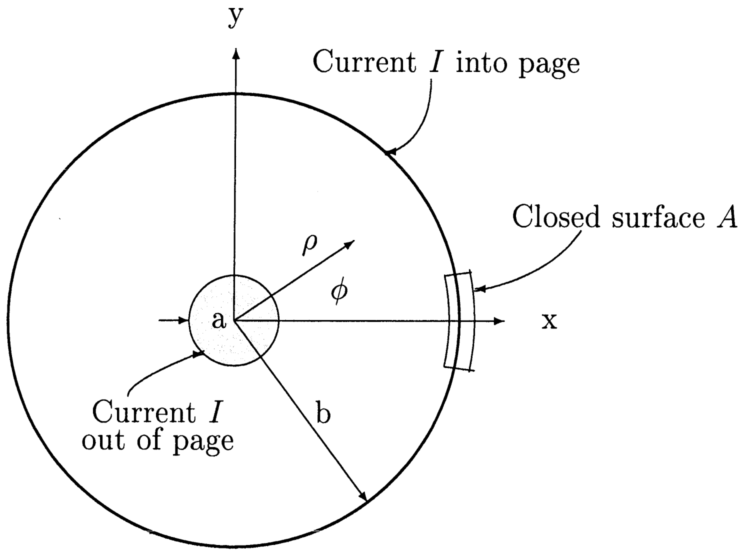
E & M Qualifier

August 17, 2005

FOR EACH PROBLEM
STATE YOUR SYSTEM OF UNITS
e.g., SI or Gaussian



1. The central conductor (radius a) of a very long coaxial cable carries a DC current I out of the page and returns this current into the page via a thin-walled-cylindrical shielding of radius $b > a$ (see Figure). **The currents are uniformly distributed over the conductors.**

   (a) [2 points] Using cylindrical coordinates ρ, ϕ, and z, determine the magnitude of the magnetic induction $\mathbf{B}$ at distances $a < \rho < b$ from the z axis (assume the wire is so long there is no z dependence).

   (b) [1 points] Determine the x and y components of $\mathbf{B}$ at a position given by coordinates (ρ, ϕ) with $a < \rho < b$.

   (c) [2 points] Using the fact that the force per unit length on a long straight wire carrying a current $\mathbf{\Delta I}$ is given by $\mathbf{\Delta F}/\Delta L \propto \mathbf{\Delta I} \times \mathbf{B}$, find an expression for the pressure (force per area) exerted by the inside wire on the outside cylindrical shielding.

   (d) [1 points] Is the pressure exerted by the inside wire inward or outward?

   (e) [1 points] Give an expression for the three components of the Maxwell stress tensor, $\vec{T^x} = T^{xx}, T^{xy}, T^{xz}$ at any point (ρ, ϕ, z) with $a < \rho < b$.

   (f) [2 points] Evaluate the integral $\int\int_A \vec{T^x} \cdot d\mathbf{a}$ to find the pressure on the outside cylinder. {The closed surface A is the small Gaussian surface whose cross-section is shown on the right of the figure. It is of finite depth in the z-direction and its' inner and outer surfaces are parallel and very close to a narrow strip of the cable's outer conductor $(\rho = b \pm \epsilon)$.}

   (g) [1 points] Explain why your answers in (f) and (c) differ.

Prob 1 (Gaussian)

(a) Using amperian loop

$$\oint \bar{B}\cdot dl = \frac{4\pi}{c} I_{enc}$$

$$B\, 2\pi\rho = \frac{4\pi}{c} I$$

$$\bar{B} = \frac{2I}{c\rho}\hat{\phi}$$

b) $$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

$$\bar{B} = \frac{2I}{c(x^2+y^2)^{1/2}}(-\sin\phi\,\hat{x} + \cos\phi\,\hat{y})$$

c) $$\frac{\Delta F}{\Delta L} \propto \Delta\bar{I}\wedge\bar{B}$$

$$\Rightarrow \frac{\Delta F}{\Delta L} = \frac{2I^2}{c\rho}(-\hat{\rho})$$

$$\Rightarrow \frac{\Delta F}{2\pi b\Delta L} = \frac{2I^2}{2\pi bc\rho}(-\hat{\rho})$$

$$\Rightarrow \bar{P} = \frac{\Delta F}{\Delta A} = \frac{I^2}{\pi bc\rho}(-\hat{\rho})$$

d) Inward since, $\overline{P} = P(-\hat{\rho})$

e) $$T^{\alpha\beta} = \frac{1}{4\pi}\left[E^\alpha E^\beta + B^\alpha B^\beta - \frac{1}{2}(E^2+B^2)\delta^{\alpha\beta}\right]$$

$$T^{xx} = \frac{1}{4\pi}\left[(B_x)^2 - \frac{1}{2}B^2\right] = \frac{1}{4\pi}\frac{4I^2}{c^2\rho^2}\left(\sin^2\phi - \frac{1}{2}\right)$$

$$T^{xy} = \frac{1}{4\pi}(B_x B_y) = -\frac{1}{4\pi}\frac{4I^2}{c^2\rho^2}\sin\phi\cos\phi$$

$$T^{xz} = \frac{1}{4\pi}B_x B_z = 0$$

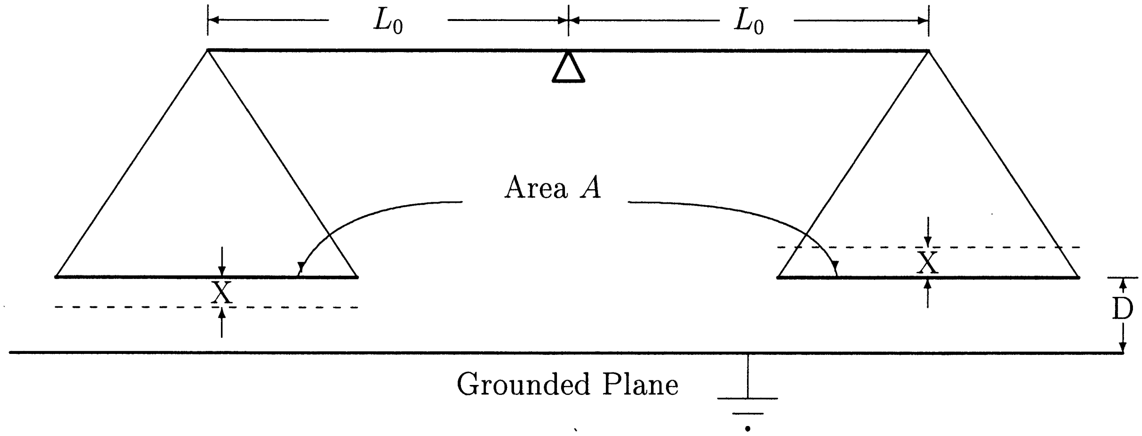
$$\vec{T}^x = \begin{pmatrix} \sin^2\phi - \frac{1}{2} \\ -\sin\phi\cos\phi \\ 0 \end{pmatrix}\frac{I^2}{\pi c^2\rho^2}$$

f) $$F = \int \vec{T}^x \cdot d\vec{a} = \int \sum_\beta T_{x\beta}\, n_\beta\, da$$

$$d\bar{a} = \rho\, d\phi\, dz\, \hat{\rho} = \rho\, d\phi\, dz \begin{pmatrix}\cos\phi \\ \sin\phi \\ 0\end{pmatrix}$$

$$\bar{F} = \int \frac{I^2}{\pi c^2\rho^2}\left(\sin^2\phi - \frac{1}{2}\right)\cos\phi\, \rho\, d\phi\, dz + \int \frac{I^2}{\pi c^2\rho^2}(-\sin\phi\cos\phi)\sin\phi\, \rho\, d\phi\, dz$$

$$F = -\frac{I^2}{2\pi c^2 \rho} \int \cos\phi \, d\phi \, dz$$



2. Two identical thin square metal plates (in vacuum) with areas A and masses M are suspended by non-conducting threads from each end of a massless rod of length $2L_0$ (see Figure). The rod is supported in the middle by a frictionless bearing so that each plate is suspended a distance D above a very large, grounded metal plane thus forming two parallel-plate capacitors. Each plate is parallel to the grounded plane and when it is uncharged the system is perfectly balanced as long as the separation between either plate and the grounded plane is less than $2D$. When each plate is a distance D from the grounded plane, its capacitance is C_0. Assume that fringing fields can be neglected in this problem.

   (a) [2 points] Find the capacitance C_L of the left-hand capacitor and the capacitance C_R of the right-hand capacitor (in terms of C_0, D, and X) when the left-hand plate is a distance $D - X$ from the grounded plate (and thus the right hand plate is a distance $D+X$ above grounded plane).

   (b) A charge Q_0 is placed on each plate (i.e. on the left hand and right hand suspended plates). Assume that the two plates are NOT electrically connected and that the left hand plate is initially displaced so that it is a distance $D - X$ from the grounded plane (assume $X << D$).

      i. [2 points] Find U, the electrical energy stored on the two capacitors, as a function of X.

      ii. [2 points] Find the net electrical force on the two plates as a function of X. Comment on the resulting motion, i.e., on $X(t)$.

   (c) The two plates are now connected together (by a massless wire) and to a battery of voltage V_0. Again the left hand plate is initially displaced so that it is a distance $D - X$ from the grounded plane (assume again $X << D$).

check ↗

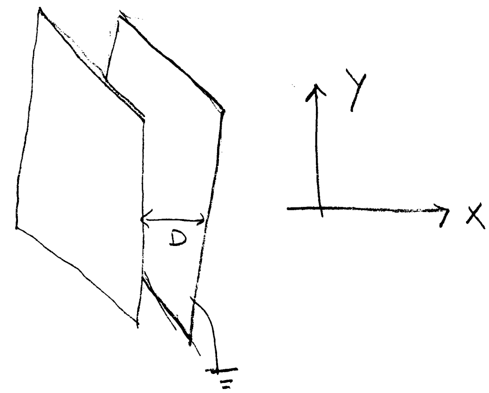
i. [1 points] Find U, the electrical energy stored on the two capacitors, as a function of X.

ii. [2 points] Find the electrical energy supplied by the battery as a function of X.

iii. [1 points] Find the net electrical force on the two plates as a function of X. Comment on the resulting motion, i.e., on $X(t)$.

Prob 2 (Gaussian)

a) $C = \frac{Q}{V}$



$E.A = 4\pi Q$

$\Rightarrow \bar{E} = 4\pi \frac{Q}{A} \hat{x}$

$\Rightarrow V = -\int_D^0 \bar{E} \cdot d\bar{r} = -ED = +4\pi ED = 4\pi \frac{QD}{A}$

$\Rightarrow C_o = \frac{A}{4\pi D}$

Now,

$V_L = -\int_{D-x}^{0} E \cdot dr = E(D-x) = \frac{4\pi Q}{A}(D-x)$

$\Rightarrow C_L = \frac{Q}{V_L} = \frac{A}{4\pi(D-x)} = \frac{C_o}{(1-\frac{x}{D})}$

$\Rightarrow C_R = \frac{C_o}{(1+\frac{x}{D})}$

b) i. $U_L = \frac{1}{8\pi} E_L^2 V = \frac{2\pi Q_0^2}{A^2} A (D-x)$

$$U_R = \frac{2\pi Q_0^2}{A}(D+x)$$

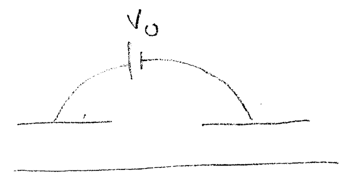
ii. $\bar{F}_L = \frac{dU_L}{dx} = -\frac{2\pi Q_0^2}{A}\hat{y}$

$$F_R = \frac{2\pi Q_0^2}{A}\hat{y}$$

} equal & opposite force

$x(t) = 0$

c) $U = \frac{1}{2} C V^2$



i) $\Rightarrow U_L = \frac{C_0 V^2}{2\left(1-\frac{x}{D}\right)} = \frac{1}{2} Q_L V$

$$\Rightarrow U_R = \frac{C_0 V^2}{2\left(1+\frac{x}{D}\right)} = \frac{1}{2} Q_R V$$

ii) $U_b = (Q_L + Q_R)V = C_0 V^2 \left\{ \frac{1}{\left(1-\frac{x}{D}\right)} + \frac{1}{1+\frac{x}{D}} \right\} = \frac{2 C_0 V^2}{\left(1-\frac{x^2}{D^2}\right)}$

iii)

3. (a) [1 point] Use Maxwell's equations to show that **E** and **B** can be written in terms of scalar and vector potentials $\Phi(\mathbf{r},t)$ and $\mathbf{A}(\mathbf{r},t)$.

(b) [1 points] In the absence of polarizable and/or magnetizable material (i.e., only free charges and currents present) show that in the Lorentz gauge, Maxwell's equations reduce to the inhomogeneous wave equation:

$$\Box\, F(\mathbf{r},t) = S(\mathbf{r},t),$$

where

$$\Box \equiv \left(\frac{\partial}{c\partial t}\right)^2 - \nabla^2$$

and F stands for the potential field and S the charge/current source. Recall that the Lorentz gauge is

$$\frac{\partial}{c\,\partial t}\Phi + \nabla\cdot\mathbf{A} = 0, \quad \text{in Gaussian units}$$

$$\frac{\partial}{c^2\,\partial t}\Phi + \nabla\cdot\mathbf{A} = 0, \quad \text{in SI units.}$$

An infinitely long thin straight and uncharged wire along the z-axis carries no current for $t < 0$, however, at $t = 0$ a constant current I_0 abruptly flows in the +z direction and remains constant for $t \geq 0$. Assume the wire remains neutral. Find the following at a point a perpendicular distance ρ from the wire for all times t:

(c) [4 points] The potentials Φ and **A**,

(d) [2 points] The fields **B** and **E**,

(e) [1 points] The Poynting vector **S**.

(f) [1 points] Show that your results for (c), (d), and (e) reduce to sensible values for $t \to \infty$.

Hint:

$$\int \frac{dX}{\sqrt{X^2 + a^2}} = \ln(\sqrt{X^2 + a^2} + X).$$

Prob 3 (Gaussian)

a)
$$\bar{\nabla}\cdot\bar{D} = 4\pi\rho_f$$
$$\bar{\nabla}\cdot\bar{B} = 0$$
$$\nabla\wedge\bar{E} = -\frac{1}{c}\frac{\partial\bar{B}}{\partial t}$$
$$\nabla\wedge\bar{H} - \frac{1}{c}\frac{\partial\bar{D}}{\partial t} = \frac{4\pi}{c}\bar{J}$$

$\rightarrow \bar{\nabla}\cdot\bar{B} = 0$

since the divergence of the curl of any vector func is zero i.e $\bar{\nabla}\cdot(\nabla\wedge\bar{A}) = 0$

$$\Rightarrow \bar{\nabla}\cdot\bar{B} = \bar{\nabla}\cdot(\bar{\nabla}\wedge\bar{A})$$

$$\Rightarrow \bar{B} = \bar{\nabla}\wedge\bar{A}$$

then, $$\nabla\wedge\bar{E} = -\frac{1}{c}\frac{\partial B}{\partial t} = -\frac{1}{c}\frac{\partial}{\partial t}(\nabla\wedge\bar{A}) = -\nabla\wedge\frac{1}{c}\frac{\partial A}{\partial t}$$

$$\Rightarrow \nabla\wedge\left(E + \frac{1}{c}\frac{\partial\bar{A}}{\partial t}\right) = 0$$

but, the curl of the gradient of a scalar func is zero i.e. $\nabla\wedge(-\nabla\Phi) = 0$

$$\Rightarrow \bar{E} = -\nabla\Phi - \frac{1}{c}\frac{\partial\bar{A}}{\partial t}$$

b) $\nabla \cdot \bar{E} = 4\pi\rho$

$$\nabla \wedge \bar{B} - \frac{1}{c}\frac{\partial E}{\partial t} = \frac{4\pi}{c}\bar{J}$$

i) $\bar{\nabla} \cdot \left(-\nabla \Phi - \frac{1}{c}\frac{\partial \bar{A}}{\partial t}\right) = 4\pi\rho$

$$\Rightarrow -\nabla^2 \Phi - \frac{1}{c}\frac{\partial}{\partial t}(\nabla \wedge \bar{A}) = 4\pi\rho$$

ii) $\nabla \wedge (\bar{\nabla} \wedge \bar{A}) - \frac{1}{c}\frac{\partial}{\partial t}\left(-\nabla \Phi - \frac{1}{c}\frac{\partial \bar{A}}{\partial t}\right) = \frac{4\pi}{c}\bar{J}$

F-05

4. A dielectric sphere of radius R (with dielectric constant ϵ) is embedded with free charge so that it acquires a polarization of $\mathbf{P} = (K/r)\,\hat{\mathbf{r}}$ where K is a given constant, r is the distance from the center of the sphere, and $\hat{\mathbf{r}}$ is the radial unit vector.

   (a) [2 points] Calculate the bound charge volume and surface densities ρ_b and σ_b respectively.

   (b) [1 point] Find the electric field $\mathbf{E}$ inside the dielectric.

   (c) [2 points] Calculate the volume density of free charge ρ_f.

   (d) [1 point] Find the electric field $\mathbf{E}$ outside the dielectric.

   (e) [4 points] Calculate the electric potential V, inside and outside the dielectric.

I don't believe/understand the solution

Prob 4 (Gaussian)

a) $\rho_b = -\bar{\nabla} \cdot \bar{P}$ where, $\bar{P} = \frac{K}{r}\hat{r}$

$$= -\frac{1}{r^2 \sin\theta} \frac{\partial}{\partial r}\left(r^2 \sin\theta \frac{K}{r}\right)$$

$$= -\frac{K}{r^2}$$

$$\sigma_b = \bar{P} \cdot \hat{r} = \frac{K}{r}$$

b) $\bar{D} = \bar{E} + 4\pi\bar{P}$

$$\Rightarrow \epsilon \bar{E} = \bar{E} + 4\pi\bar{P}$$

$$\Rightarrow \bar{E} = \frac{4\pi\bar{P}}{\epsilon - 1} = \frac{4\pi K}{(\epsilon - 1) r}\hat{r}$$

c) $\bar{D} = \frac{\epsilon 4\pi K}{(\epsilon - 1) r}\hat{r}$

~~$\oint \bar{D} \cdot d\bar{a} = 4\pi Q_{f\,enc}$~~

~~$\Rightarrow D(4\pi r^2) = 4\pi Q_{f\,enc}$~~

$\nabla \cdot \bar{D} = 4\pi\rho_f$

$$\Rightarrow \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial r}\left(r^2 \sin\theta \frac{1}{r}\right) \frac{4\pi K}{(1 - \frac{1}{\epsilon})} = 4\pi\rho_f$$

$$\Rightarrow \rho_f = \frac{K}{(1 - \frac{1}{\epsilon}) r^2}$$

~~$\Rightarrow Q_f \neq \frac{\epsilon 4\pi K r}{(\epsilon - 1)}$~~

~~$\Rightarrow \rho_f = \frac{3\epsilon K}{(\epsilon - 1) r^2}$~~

a)

$$\oint \bar{D}\cdot d\bar{a} = 4\pi \int \rho_f \, d^3x$$

$$\Rightarrow D\, 4\pi r^2 = 4\pi \int \frac{K}{(1-\frac{1}{\epsilon})r^2} r^2 \sin\theta \, d\theta \, d\phi$$

$$\Rightarrow D = \frac{4\pi K R}{(1-\frac{1}{\epsilon})r^2} \Rightarrow \bar{E} = \frac{4\pi K R}{(\epsilon-1)r^2} \checkmark$$

$\nRightarrow$

$$\int \bar{E}\cdot d\bar{a} = 4\pi \int \rho_{tot} \, d^3x$$

$$= 4\pi \int (\rho_f + \rho_b)\, d^3x$$

$$= 4\pi \int \left(-\frac{K}{r^2} + \frac{K}{(1-\frac{1}{\epsilon})r^2}\right) r^2 dr \sin\theta \, d\theta \, d\phi$$

$$= (4\pi)^2 \frac{-K(1-\frac{1}{\epsilon}) + K}{(1-\frac{1}{\epsilon})} R$$

$$= (4\pi)^2 \frac{KR}{\epsilon(1-\frac{1}{\epsilon})}$$

$$= (4\pi)^2 \frac{KR}{(\epsilon-1)}$$

$$E = \frac{4\pi K R}{(\epsilon-1)r^2} \checkmark$$

e) • outside

$$V = -\int_{\infty}^{r} \bar{E}\cdot d\bar{r} = -\int_{+\infty}^{r} \frac{4\pi KR}{(\epsilon-1)} \frac{1}{r^2} dr$$

$$= \frac{4\pi KR}{(\epsilon-1)} \left[\frac{1}{r}\right]_{\infty}^{r}$$

$$= + \frac{4\pi KR}{(\epsilon-1)r}.$$

• inside

$$V = -\int_{\infty}^{r} \bar{E}\cdot d\bar{r} = -\int_{+\infty}^{R} \bar{E}\cdot d\bar{r} - \int_{R}^{r} \bar{E}\cdot d\bar{r}$$

$$= -\int_{\infty}^{R} \frac{4\pi KR}{(\epsilon-1)} \frac{1}{r^2} dr - \int_{R}^{r} \frac{4\pi K}{(\epsilon-1)} \frac{1}{r} dr$$

$$= \frac{4\pi KR}{(\epsilon-1)} \left[\frac{1}{r}\right]_{\infty}^{R} - \frac{4\pi K}{(\epsilon-1)} \ln r \Big|_{R}^{r}$$

$$= \frac{4\pi KR}{(\epsilon-1)} \frac{1}{R} - \frac{4\pi K}{(\epsilon-1)} \ln\left(\frac{r}{R}\right) = \frac{4\pi K}{\epsilon-1} \left[1 - \ln\left(\frac{r}{R}\right)\right]$$

F_05

5. Two charges of equal magnitude but opposite signs are symmetrically placed on the z axis at $z = \pm d$. A grounded conducting sphere of radius a ($a < d$) is also centered on the origin. The total potential for $r > a$ can be written as the sum of 3 terms,

$$\Phi_{total} = \Phi_{sphere} + \Phi_{+q} + \Phi_{-q},$$

respectively, the potentials due to the sphere, the $+q$ charge and the $-q$ charge.

(a) [2 points] Give an expression for $\Phi_{sphere}(r, \theta)$ valid for $r > a$, as a sum over Legendre polynomials having arbitrary constant coefficients. Make sure you potential vanishes at ∞.

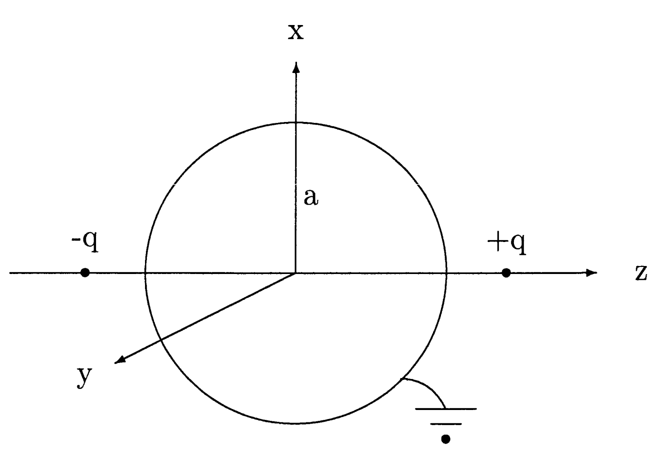
(b) [2 points] Give two expressions for $\Phi_q(r, \theta)$ as a sums over Legendre polynomials, one valid for $a < r < d$ and one valid for $d < r$. (Explicitly give the coefficients.)

(c) [2 points] Give two expressions for $\Phi_{-q}(r, \theta)$ as a sums over Legendre polynomials, one valid for $a < r < d$ and one valid for $d < r$. Explicitly give the coefficients.

(d) [1 points] Give two expressions for $\Phi_q + \Phi_{-q}$ as a sums over Legendre polynomials one valid for $a < r < d$ and one valid for $d < r$. Explicitly give the coefficients.

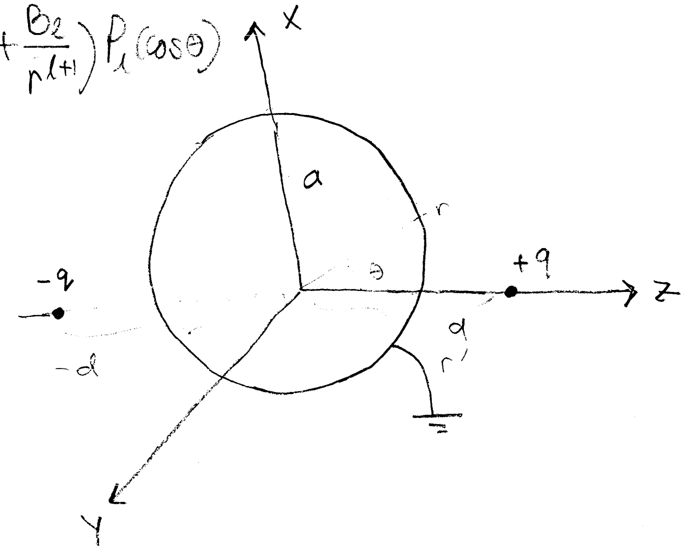
(e) [2 points] Combine the above and evaluate the unknown constants of part (a) by insisting that $\Phi_{total}(r = a, \theta) = 0$.

(f) [1 points] Give two expressions for $\Phi_{total}(r, \theta)$ as a sums over Legendre polynomials, one valid for $a < r < d$ and one valid for $d < r$. Explicitly give the coefficients.



Prob 5 (Gaussian)

a) $\nabla^2 \Phi_{sphere} = 0 \Rightarrow \Phi_{sphere}(r,\theta) = \sum_{l} \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l(\cos\theta)$



for $r > a$

$$\Phi_{sphere}(r,\theta) = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta)$$

b) $$\Phi_q(r,\theta) = \frac{q}{|\bar{r}-\bar{r}'|} = q \sum_{l=0}^{\infty} \frac{r_<^l}{r_>^{l+1}} P_l(\cos\theta)$$

$$\Phi_q(r,\theta)\Big|_{r>d} = q \sum_{l=0}^{\infty} \frac{d^l}{r^{l+1}} P_l(\cos\theta)$$

$$\Phi_q(r,\theta)\Big|_{a<r<d} = q \sum_{l=0}^{\infty} \frac{r^l}{d^{l+1}} P_l(\cos\theta)$$

c) $$\Phi_{-q}(r,\theta)\Big|_{r>d} = -q \sum_{l=0}^{\infty} \frac{d^l}{r^{l+1}} \underbrace{P_l(\cos(\pi-\theta))}_{P_l(-\cos\theta)}$$

$$= -q \sum_{l=0}^{\infty} \frac{(-1)^l d^l}{r^{l+1}} P_l(\cos\theta)$$

$$\Phi_{-q}(r,\theta)\Big|_{a<r<d} = -q \sum_{l=0}^{\infty} \frac{(-1)^l r^l}{d^{l+1}} P_l(\cos\theta)$$

d)

$$\left(\Phi_q(r,\theta) + \Phi_{-q}(r,\theta)\right)\Big|_{r>d} = q\sum_{l=0}^{\infty} \frac{d^l}{r^{l+1}} P_l(\cos\theta)\left[1-(-1)^l\right]$$

$$\left(\Phi_q(r,\theta) + \Phi_{-q}(r,\theta)\right)\Big|_{a<r<d} = q\sum_{l=0}^{\infty} \frac{r^l}{d^{l+1}} P_l(\cos\theta)\left[1-(-1)^l\right]$$

e)

$$\Phi_{tot}(r=a,\theta) = 0$$

$$\leadsto \quad \Phi_{sphere}(r=a,\theta) + \left(\Phi_q(r,\theta) + \Phi_{-q}(r,\theta)\right)_{a<r<d} = 0$$

$$\Rightarrow \sum_{l=0}^{\infty} \left\{ \frac{B_l}{a^{l+1}} + q\frac{a^l}{d^{l+1}}\left(1-(-1)^l\right)\right\} P_l(\cos\theta) = 0$$

$$\Rightarrow \quad B_l = -\frac{q a^{2l+1}}{d^{l+1}}\left(1-(-1)^l\right)$$

$\Rightarrow l$ has to be odd

f)

$$\Phi_{tot}(r,\theta)\Big|_{r>d} = \sum_{\substack{l=0 \\ l \text{ odd}}}^{\infty} \left(-\frac{2q a^{2l+1}}{d^{l+1} r^{l+1}} + \frac{2q d^l}{r^{l+1}}\right) P_l(\cos\theta)$$

$$\Phi_{tot}(r,\theta)\Big|_{a<r<d} = \sum_{l=0}^{\infty} \left(-\frac{2q\, a^{2l+1}}{d^{l+1} r^{l+1}} + \frac{2q r^l}{d^{l+1}}\right) P_l(\cos\theta)$$

6. A monochromatic plane electromagnetic wave, polarized in the x-direction and traveling in a vacuum in the z-direction in the Lab can be written in the following 3 dimensional form:

$$\vec{E} = E_0 \ \hat{\imath} \ e^{ik(z-ct)},$$

$$\vec{B} = B_0 \ \hat{\jmath} \ e^{ik(z-ct)},$$

where $\hat{\imath}, \hat{\jmath}, \hat{k}$ are unit vectors in the respective x, y, z directions and k is real. In Gaussian units $\epsilon = \mu = 1$ and $B_0 = E_0$.

(a) [1 point]
What is the significance of $\vec{E}$ and $\vec{B}$ being complex?

(b) [2 points]
Combine the above electric and magnetic fields, $\vec{E}$ and $\vec{B}$, into a single electromagnetic field 4-tensor $F^{\alpha\beta}$, (where $\alpha, \beta = 0, 1, 2, 3$ and $x^\alpha = (ct, x, y, z)$).

(c) [3 points]
Use a Lorentz boost (i.e., a constant velocity transformation) to find $F'^{\alpha\beta}$ in a frame moving in the Lab's $+z$ direction with a speed v. Don't forget to express your answer in terms of the moving coordinates $x'^\alpha = (ct', x', y', z')$.

(d) [2 points]
From your results for the previous part, give the frequency and the wavelength of the wave in the moving frame, in terms of their Lab values.

(e) [2 points]
How have $\vec{E}'$ and $\vec{B}'$ changed in direction and magnitude?

6.

$$\bar{E} = E_0 \, e^{ik(z-ct)} \, \hat{x}$$