Zero-range Interacting Atoms in Low-dimensional Harmonic Traps

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- Broadly speaking, magneto-optical traps trap and cool atoms
- For small deviations from the center of the trap, the trapping potential can be modelled as a simple harmonic oscillator

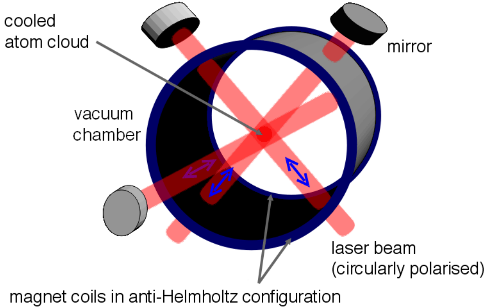


Figure 1: Image taken from Wikipedia

- Magneto-optical traps allow us to study:
  - Dynamics and equilibration
  - Interplay between quantum fluctuations and thermal fluctuations
  - Effectively 1D systems
  - Tunable interactions by varying the magnetic field in an optical trap
- For our purposes, we will focus on effectively 1D systems with tunable interactions
- Bottom-up approach to few-to-many body physics

1D Simple Harmonic Oscillator

- $\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + V(z)\right)\psi(z) = E\psi(z)$
- $V(z) = \frac{1}{2}m\omega_z^2 z^2$
- Can experimentally be realized by taking $\omega_x,\ \omega_y \gg \omega_z$

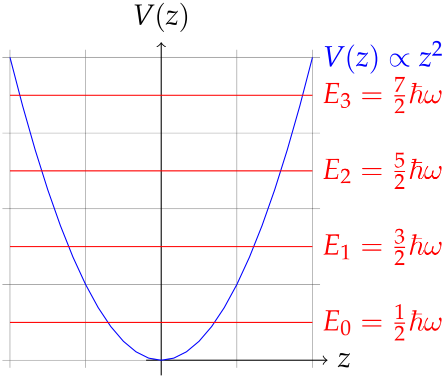


Figure 2: Discrete energy levels of simple harmonic oscillator

Dynamics in a Magneto-Optical Trap

- How can we understand the dynamics of systems confined to a magneto-optical trap?
  - Interactions?
  - Exchange statistics?
- Look at simple cases: two identical bosons and two identical fermions in 1D

Zero-range Interaction between Two Identical Bosons

- By exchange symmetry, we investigate only even-parity solutions to the time-independent Schrödinger equation
- Zero-range interaction is given by $V^{+}(z) = g_{+}\delta(z)$
- Solving $(H_{HO} + V^{+})\psi = E\psi$ using a Green's function approach, we obtain the following energy spectrum:

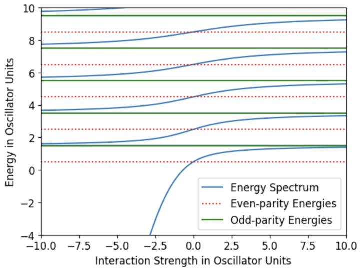


Figure 3: Agrees with K. Kanjilal and D. Blume. Phys. Rev. A **70**, 042709 (2004).

Zero-range Interaction between Two Identical Fermions

- Similar approach as before, with exchange antisymmetry and interaction $V^{-}(z) = g_{-}\frac{\delta(z)}{z}\frac{\partial}{\partial z}$

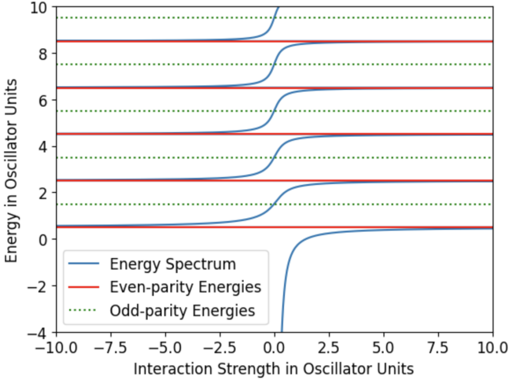
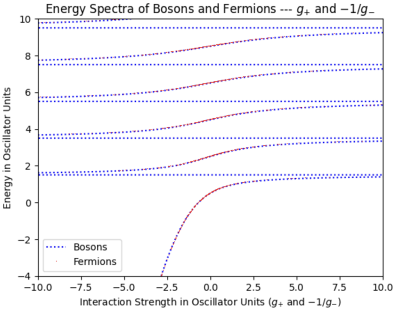
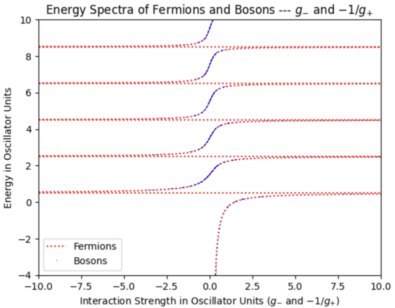


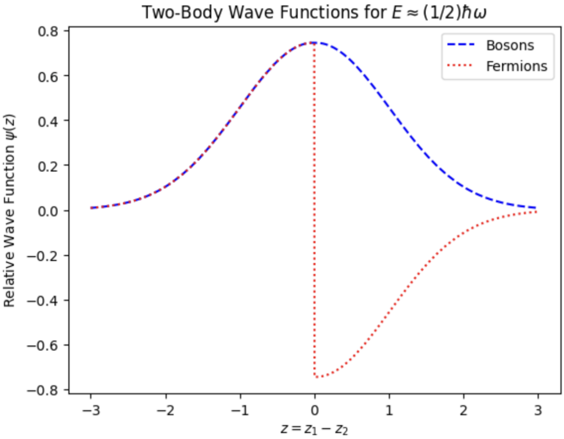
Figure 4: Agrees with K. Kanjilal and D. Blume. Phys. Rev. A **70**, 042709 (2004).

- Weakly-interacting fermions act like strongly-interacting bosons and vice versa
- Same energies, but different wavefunctions





- Same energies, but different wavefunctions



2 Spin-up Fermions, 1 Spin-down Fermion

- Only have interactions between spin-up and spin-down
  - Interaction $V_{int} = g_+ [\delta(z_1 - z_3) + \delta(z_2 - z_3)]$
  - Spin-up particles have coordinates z_1, z_2, resp., and spin-down particle has coordinate z_3

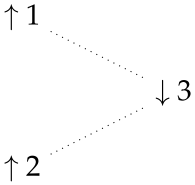


Figure 5: Zero-range interactions

- We can derive the following equation for the relative eigenenergies:

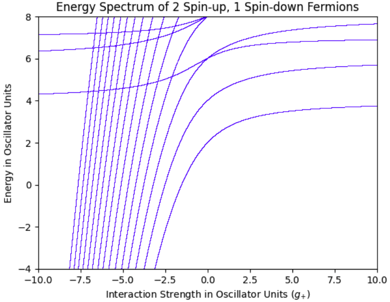
$$\begin{bmatrix} A_{0,0} & \dots & A_{0,k} \\ \vdots & \ddots & \vdots \\ A_{k,0} & \dots & A_{k,k} \end{bmatrix} \vec{v} = \frac{1}{g_+} \vec{v},$$

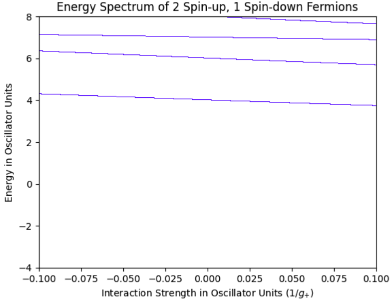
where

$$A_{i,j} = \int_{-\infty}^{\infty} \left(\phi_i^*(z) \phi_j(-\frac{1}{2}z) G_{2B}(E_{3B} - E_j,\ -\frac{\sqrt{3}}{2}z,\ 0) \right)\ dz \\ - \delta_{i,j} G_{2B}(E_{3B} - E_j,\ 0,\ 0)$$

2 Spin-up, 1 Spin-down Fermion Energy Spectrum

- Wrote code to numerically solve the eigenvalue equation and determine the eigenenergies for negative relative parity





- We have a few sanity checks:
  - Expected non-interacting limit relative energies
  - Expected strongly-interacting limit relative energies
  - Perturbative energy shifts corresponding to slopes near the non-interacting and strongly-interacting limits

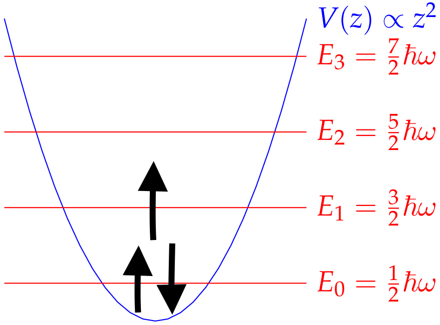
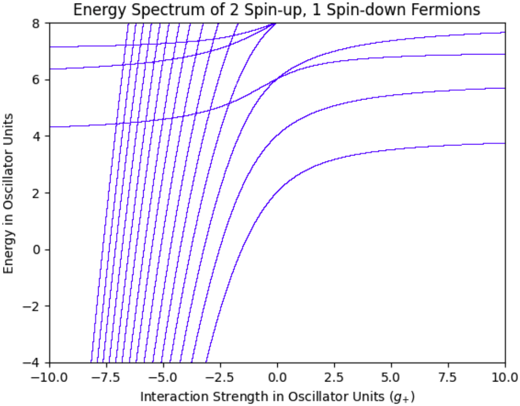


Figure 6: Ground state of 2F1F′ system with no interaction

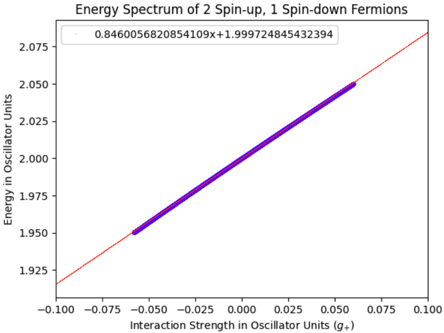
- We have a few sanity checks:
    - Expected non-interacting limit relative energies of $2\hbar\omega, 4\hbar\omega, 6\hbar\omega, 8\hbar\omega, \ldots$ with degeneracies of $1, 1, 2, 3, \ldots$



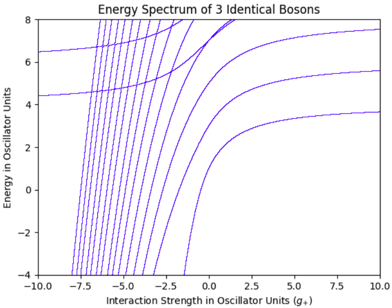
CHECKING THESE RESULTS

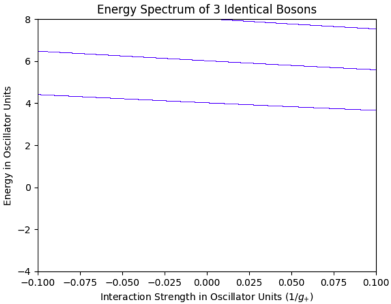
- First order non-degenerate perturbation theory gives that for the state in the non-interacting limit with relative energy $2\hbar\omega$, one expects a perturbative shift of

$$\Delta E = g_+ \int \left(\phi_1(z_{12})\phi_0(z_{12,3})\right)^* \left(\delta(z_{13}) + \delta(z_{23})\right) \phi_1(z_{12})\phi_0(z_{12,3}) d\mathbf{z}$$
$$= \frac{3}{2\sqrt{\pi}} g_+ \approx 0.846 g_+$$



- A similar analytical derivation and numerical implementation gives the following spectra:

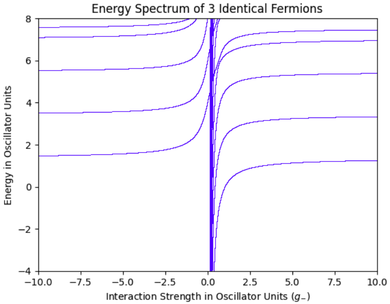


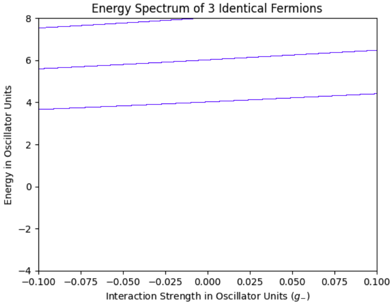


- Again, these are consistent with the “sanity checks”

Three Identical Fermions

- A similar analytical derivation and numerical implementation gives the following spectra (work-in-progress):





- Some features agree with our expectations, so the exact issue is difficult to identify

- The Fermi-Bose duality also holds between 3 identical bosons and 3 identical fermions!

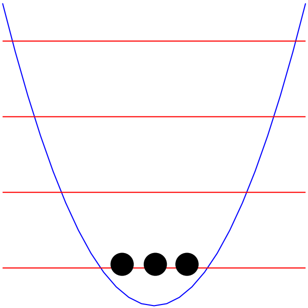


Figure 7: Bosonic picture

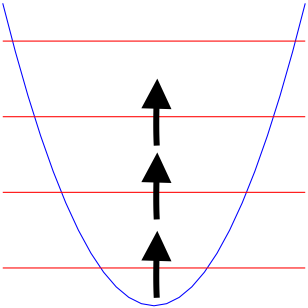


Figure 8: Fermionic picture

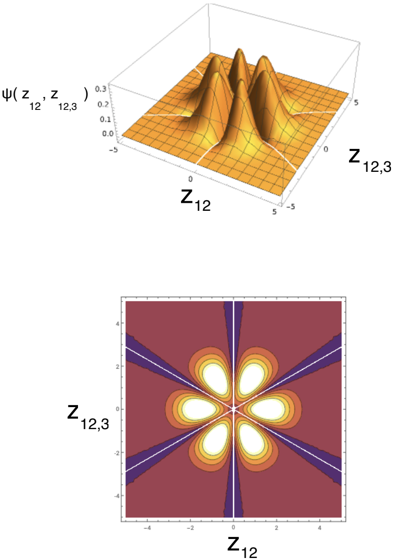


Figure 9: Bosonic picture

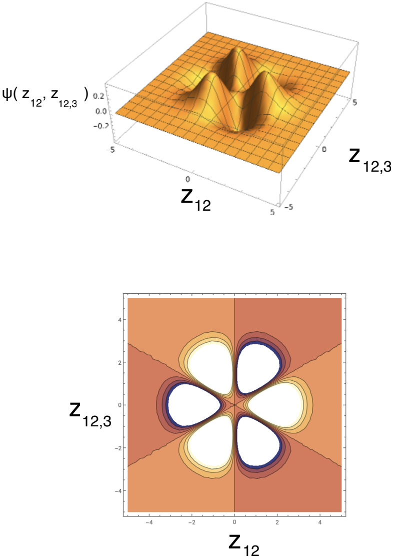


Figure 10: Fermionic picture

Future Work

- Continue work on 3F relative energy spectrum
- Move on to systems with four particles
- Adapt techniques to anyons

Acknowledgements

- Dr. Doerte Blume
- Homer L. Dodge Department of Physics and Astronomy