Zero-range Interacting Atoms in Low-dimensional Harmonic Traps

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Magneto-optical Traps

- Broadly speaking, magneto-optical traps trap and cool atoms
- For small deviations from the center of the trap, the trapping potential can be modelled as a simple harmonic oscillator

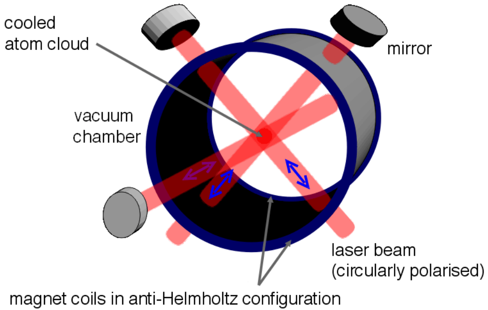

Figure 1: Image taken from Wikipedia

1D Simple Harmonic Oscillator

- $\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + V(z)\right)\psi(z) = E\psi(z)$
- $V(z) = \frac{1}{2}m\omega_z^2 z^2$
- Can experimentally be realized by taking $\omega_x,\ \omega_y \gg \omega_z$

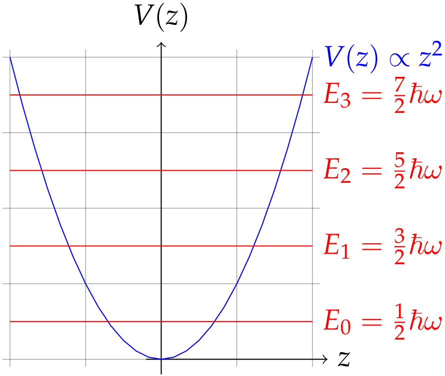

Figure 2: Discrete energy levels of simple harmonic oscillator

Dynamics in a Magneto-Optical Trap

- How can we understand the dynamics of systems confined to a magneto-optical trap?
  - Interactions?
  - Exchange statistics?
- Look at simple cases: two identical bosons and two identical fermions in 1D

Zero-range Interaction between Two Identical Bosons

- By exchange symmetry, we investigate only even-parity solutions to the time-independent Schrödinger equation
- Zero-range interaction is given by $V^{+}(z) = g_{+}\delta(z)$
- Solving $(H_{HO} + V^{+})\psi = E\psi$ using a Green's function approach, we obtain the following energy spectrum:

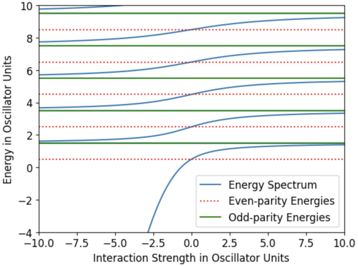

Figure 3: Agrees with K. Kanjilal and D. Blume. Phys. Rev. A **70**, 042709 (2004).

Zero-range Interaction between Two Identical Fermions

- Similar approach as before, with exchange antisymmetry and interaction $V^{-}(z) = g_{-}\frac{\delta(z)}{z}\frac{\partial}{\partial z}$

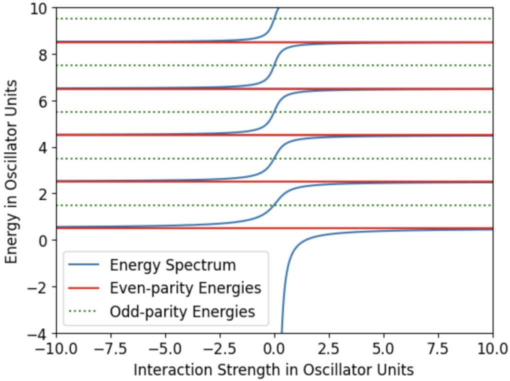

Figure 4: Agrees with K. Kanjilal and D. Blume. Phys. Rev. A **70**, 042709 (2004).

Future Work

- Generalize this approach to three- and four-particle systems with bosonic, fermionic, and mixed exchange statistics
- Already have tentative success for system of two spin-up fermions and one spin-down fermion in 1D
    - Agrees with S. E. Gharashi, K. M. Daily, and D. Blume. Phys. Rev. A **86**, 042702 (2012).

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