Manifestations of Quantum Geometric Effects

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Parallel Transport and Anholonomy

- Two vectors are displaced along paths drawn on a sphere & maintain a constant angle with the surface *locally*.
- The vectors in the figure "rotate" with respect to *each other*, but are parallel to the sphere during transport.

This global rotation without any accompanying local rotation is a result of the intrinsic curvature of the sphere.

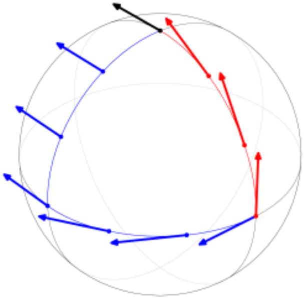

Figure: Parallel transport

Background

- **1931** — Dirac writes about a path-dependent non-integrable phase in a paper titled *"Quantised Singularities in the Electromagnetic Field"* [1].
- **1980** — Provost & Vallee introduce a Riemannian metric structure on the manifold of quantum states [2].
- **1984** — M. Berry proposes the quantum geometric phase in a seminal paper [3].

Quantum Geometric Tensor

- The state of a quantum system is described by a ray (i.e., equivalence class of vectors which differ only by a phase) in a complex (projective) Hilbert space, $\mathbf{P}(\mathcal{H})$.
- For $\mathcal{H} = \mathcal{H}_n$ finite-dimensional, the space $\mathbf{P}(\mathcal{H}_n)$ can equivalently be treated as a complex projective space: $\mathbf{P}(\mathcal{H}_n) = \mathbb{C}\mathbf{P}^{n-1}$ and structured as a smooth manifold.
- $\mathbb{C}\mathbf{P}^{n-1}$ is naturally endowed with the Fubini-Study metric — this carries mutually compatible complex, Riemannian, and symplectic structures.

Quantum Geometric Tensor

The naturally emerging metric structure is the Quantum Geometric Tensor (QGT).

$$T_{ij}\left(u_\lambda\right) \equiv \left\langle \partial_i u_\lambda \,\middle|\, \left(1 - \left|u_\lambda\right\rangle \left\langle u_\lambda\right|\right) \,\middle|\, \partial_j u_\lambda \right\rangle$$

($\partial_i = \partial/\partial(k_i)$ with $i = x, y$ & u_λ is a wave function parametrized by λ).

The QGT is simply the Fubini-Study metric on the $\mathbb{C}\mathbf{P}^{n-1}$ manifold (equivalently, the projective Hilbert space $\mathbf{P}(\mathcal{H}_n)$).

Quantum Geometric Tensor

Quantum states are represented by complex functions; hence, the QGT is complex.

$$T_{ij} \equiv g_{ij} + \frac{i}{2}\Omega_{ij}$$

- The real (symmetric) part, g_{ij}, is the quantum metric.
- The imaginary (antisymmetric) part, Ω_{ij}, is the Berry curvature.

Electronic Band Structure

The electronic band structure describes the allowed range of energy levels that electrons may occupy.

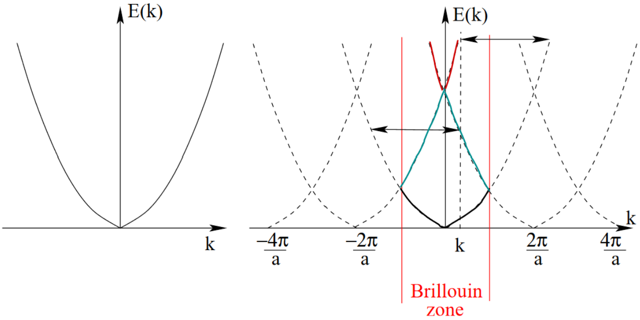

Figure: Free space eigenstates.

What are flat electronic bands?

$$E(\mathbf{k}) = \frac{\hbar^2 k^2}{2m_*}$$

$\mathbf{k}$	wavevector
m_*	effective mass

The energy range spanned by the entire band is called the bandwidth Δ.

Flat bands are systems where the energy of single electrons energy *does not depend on momentum.*

- $m_* \to \infty$
- zero velocity
- electrons localized
- Fermi surface not well defined
- $\Delta \to 0.$

Why are flat electronic bands interesting?

- In typical electronic band structures, we assume the electron interaction energy scale is *much smaller than* Δ.
  - Hence, electron-electron interactions are usually neglected.
- In a flat electronic band, $\Delta \to 0$.
  - Electron interactions can become the dominant interactions in the system.

WHY ARE FLAT ELECTRONIC BANDS INTERESTING?

Quantum geometric effects become prominent in systems with flat electronic bands.

- Lower bound for stiffness in superconductors.
- Transport despite no group velocity.
- High superconducting critical temperature (twisted bilayer graphene).

Plasmons

- **1900** — Paul Drude develops the Drude model of electrical conduction [4].
  - **1933:** Sommerfeld & Bethe incorporate quantum mechanics into the Drude model.

Early models treat interactions within a collection of electrons based on a free particle approximation.

However, this approximation mostly neglects the long-range correlations of electron positions which result from Coulomb interactions.

Plasmons

- **1952** — David Bohm & David Pines develop a “collective” treatment of interactions in a collection of electrons, describing long-range Coulomb interactions of electrons in terms of collective fields which represent organized so-called “plasma” oscillations of the whole system [5–8].
  - **1956** — David Pines coins the term “plasmon” in a 1956 review article [9].

Plasmons

More recently (2021), it has been demonstrated that there may be nontrivial internal structure within plasmon wave functions; moreover, that this may be intimately tied to their quantum geometry [10].

- In [10], the authors considered the effects of the Berry curvature — the imaginary (antisymmetric) part of the QGT.

Future Research

Goal: Investigate manifestations of quantum geometric effects in plasmons.

- *Challenge:* finding flat band systems where plasmons occur.
- In particular, how does the quantum metric manifest in plasmons?

Thank you!

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