Saturated Absorption Spectroscopy of ^{87}Rb D2 Line

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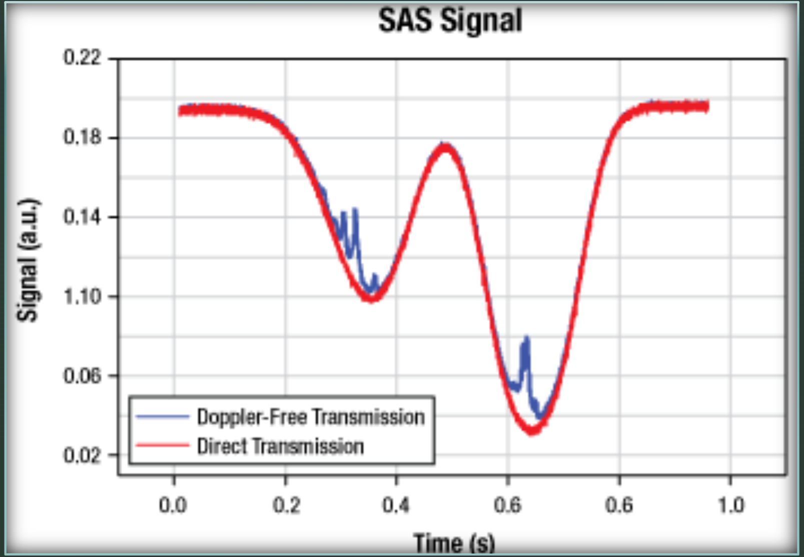
Recap

- Big picture: creating Bose-Einstein Condensates (BECs)
- Physical setup: two laser beams going through vapor cell of ^{87}Rb – **light-matter interaction**
- Want: absorption spectrum for ^{87}Rb D2 line
- Problem: doppler broadening (red)
- Solution: saturated absorption spectroscopy (blue)



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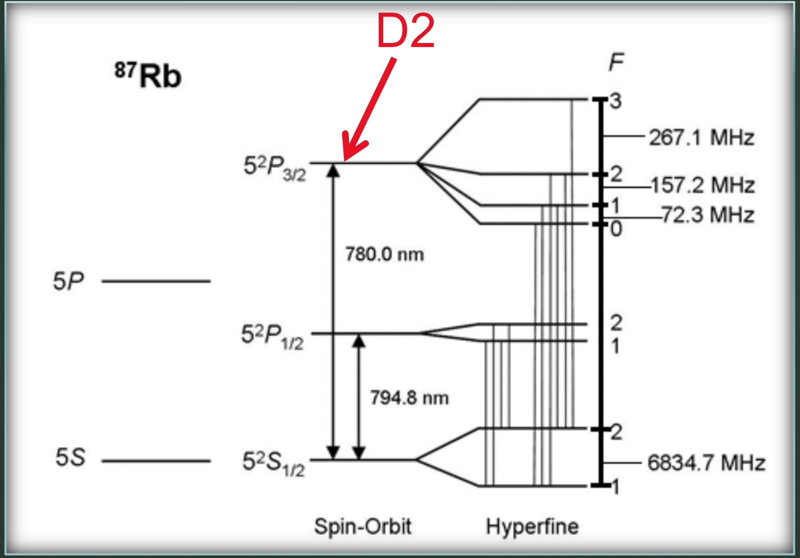

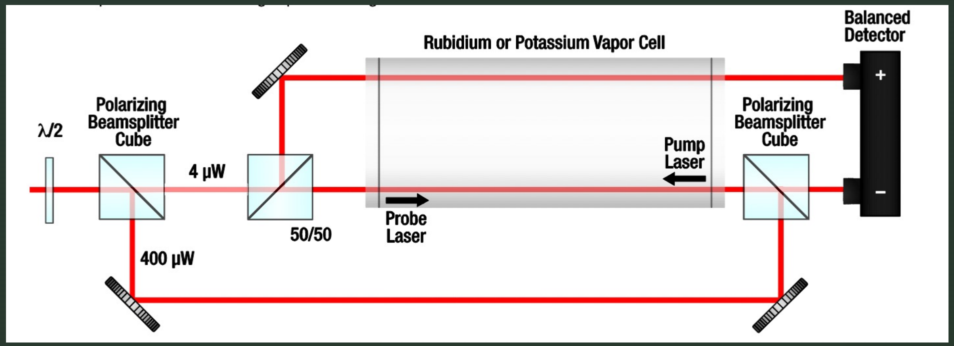

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My Project

- Multiple parameters contribute to the red signal, including temperature, velocity, atomic species, cell length, multiple transitions
- Goal: write code to plot theoretical absorption spectrum for ^{87}Rb D2 line based on different parameters
  - Can be helpful, e.g., to determine the best temperature for maximum absorption
- Requires understanding the theory behind it…

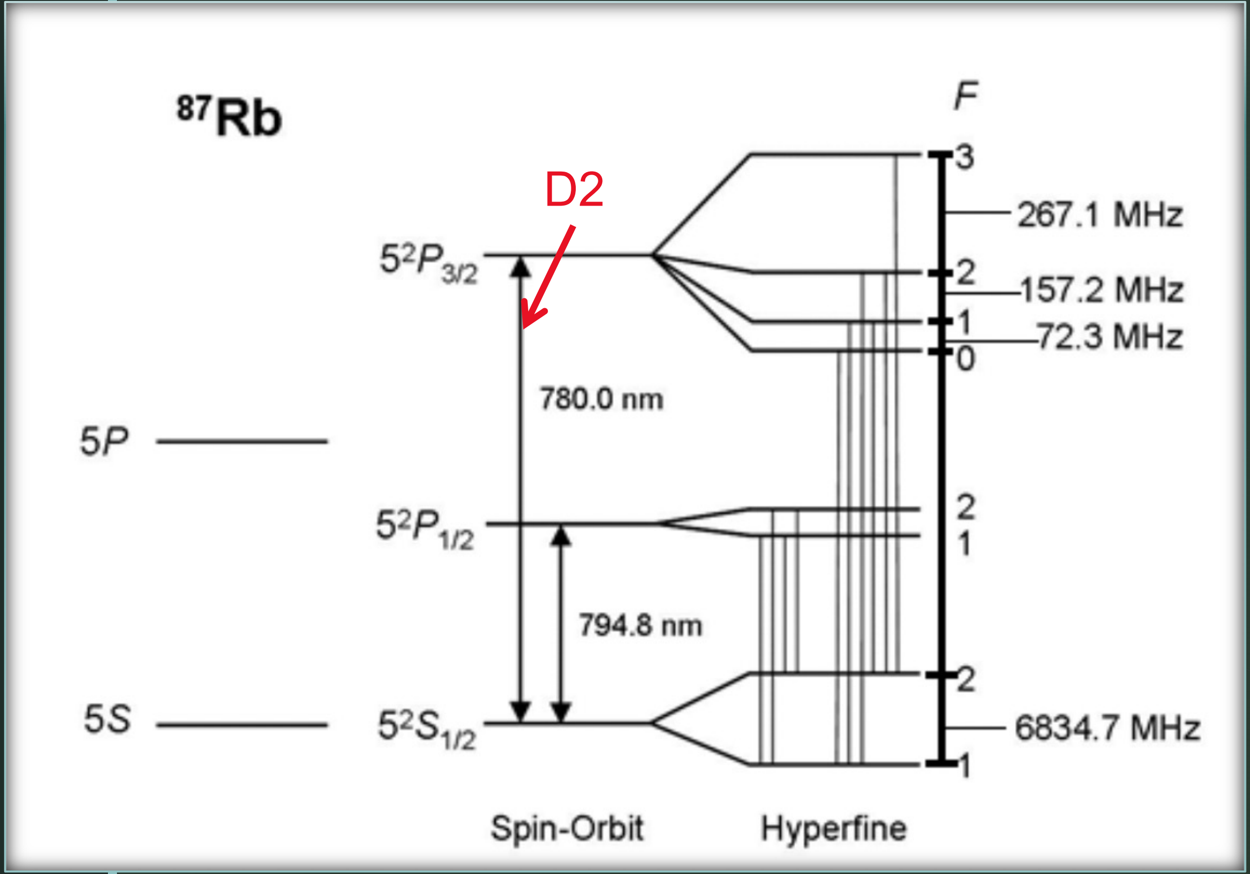
Principal Equation

$$\mathcal{T} = \exp[-(\alpha L)]$$

- $\mathcal{T}$: transmission
- **α: absorption coefficient**
- L: length of vapor cell

D2 transition: $5^2S_{1/2} \rightarrow 5^2P_{3/2}$

$n^{2S+1}L_J$



Quantum Number	Symbol	Values for $5^2S_{1/2} \rightarrow 5^2P_{3/2}$
Principal QN	n	5
Orbital Angular Momentum QN	**L**	0 → 1
Spin Angular Momentum QN	**S**	½
Total Electron Angular Momentum	**J**	½ → $^3/_2$
Total Nuclear Angular Momentum	**I**	$^3/_2$
Total Atomic Angular Momentum QN	**F**	1, 2 → 0, 1, 2, 3
Magnetic QN	m_F	$-1, 0, 1$ $-2, -1, 0, 1, 2$ $\ldots$

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Dipole Matrix Element

- Characterizes the strength of an interaction between states $|F_g, m_{F_g}\rangle$ and $|F_e, m_{F_e}\rangle$
- $\langle F_g, m_{F_g}|er_q|F_e, m_{F_e}\rangle =$

$$(-1)^{2F_e+I+J_g+J_e+L_g+S+m_{F_g}+1}\langle L_g\|\boldsymbol{er}\|L_e\rangle\sqrt{(2F_g+1)(2F_e+1)(2J_g+1)(2J_e+1)(2L_g+1)}$$

$$\begin{pmatrix} F_e & 1 & F_g \\ m_{F_e} & q & -m_{F_g} \end{pmatrix}\begin{Bmatrix} J_g & J_e & 1 \\ F_e & F_g & I \end{Bmatrix}\begin{Bmatrix} L_g & L_e & 1 \\ J_e & J_g & S \end{Bmatrix}$$

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Transition Strength

$$\left\langle F_g, m_{F_g} \middle| er_q \middle| F_e, m_{F_e} \right\rangle = c_{m_F} \langle L_g \| e\boldsymbol{r} \| L_e \rangle \equiv c_{m_F} d$$

- c_{m_F} – coefficient that determines strength of specific transition
  - Depends on initial and final states
- $\langle L_g \| e\boldsymbol{r} \| L_e \rangle \equiv d$ – reduced dipole matrix element
  - Same for all hyperfine transitions for D2 line
- $c_{m_F}^2 d^2$ – transition strength
- $C_F^2 = \Sigma c_{m_F}^2$

Breaking It Down

Quantum Number	Symbol
Principal QN	n
Orbital Angular Momentum QN	**L**
Spin Angular Momentum QN	**S**
Total Electron Angular Momentum	**J**
Total Nuclear Angular Momentum	**I**
Total Atomic Angular Momentum QN	**F**
Magnetic QN	m_F

Wigner-Eckart Theorem:

$$\left\langle F_g, m_{F_g} \middle| e r_q \middle| F_e, m_{F_e} \right\rangle = \underbrace{\langle F_g \| e\boldsymbol{r} \| F_e \rangle}_{\text{Reduced matrix element}} \underbrace{\left\langle F_g, m_{F_g} \middle| F_e, m_{F_e}; 1q \right\rangle}_{\text{Clebsch-Gordan coefficient}}$$

$$\left\langle F_g, m_{F_g} \middle| e r_q \middle| F_e, m_{F_e} \right\rangle = \underbrace{\langle F_g \| e\boldsymbol{r} \| F_e \rangle}_{\text{Depends on F, L, S, J}} \underbrace{(-1)^{F_e - 1 + m_{F_g}}}_{\text{Phase factor}} \underbrace{\sqrt{(2F_g + 1)}}_{\text{Normalization}} \underbrace{\begin{pmatrix} F_e & 1 & F_g \\ m_{F_e} & q & -m_{F_g} \end{pmatrix}}_{\text{Wigner 3-j symbol}}$$

$$\langle F_g \| e\boldsymbol{r} \| F_e \rangle \equiv \langle J_g\ I\ F_g \| e\boldsymbol{r} \| J_e\ I\ F_e \rangle = \underbrace{\langle J_g \| e\boldsymbol{r} \| J_e \rangle}_{\text{Depends on L, S, J}} \underbrace{(-1)^{F_e + J_g + 1 + I}}_{\text{Phase factor}} \underbrace{\sqrt{(2F_e + 1)(2J_g + 1)}}_{\text{Normalization}} \underbrace{\begin{Bmatrix} J_g & J_e & 1 \\ F_e & F_g & I \end{Bmatrix}}_{\text{Wigner 6-j symbol}}$$

$$\langle J_g \| e\boldsymbol{r} \| J_e \rangle \equiv \langle L_g\ I\ J_g \| e\boldsymbol{r} \| L_e\ I\ J_e \rangle = \underbrace{\langle L_g \| e\boldsymbol{r} \| L_e \rangle}_{\text{Depends on L}} \underbrace{(-1)^{J_e + L_g + S + 1}}_{\text{Phase factor}} \underbrace{\sqrt{(2J_e + 1)(2L_g + 1)}}_{\text{Normalization}} \underbrace{\begin{Bmatrix} L_g & L_e & 1 \\ J_e & J_g & S \end{Bmatrix}}_{\text{Wigner 6-j symbol}}$$

Electric Susceptibility $\chi(\Delta)$

How do I characterize a laser beam passing through an atomic vapor?

$$\chi_{F_g F_e}(\Delta) = C_F^2 d^2 \mathcal{N} \frac{1}{\hbar \epsilon_0} f_\Gamma(\Delta)$$

$C_F^2 d^2$ - transition strength of hyperfine transition
$\mathcal{N}$ – number density
$f_\Gamma(\Delta)$ - lineshape factor
Γ – decay rate

- $\chi(\Delta)$ usually complex function (atomic dipoles not in phase with driving light field)
  - Real part: characterizes dispersion
  - Imaginary part: characterizes absorption
- For atoms with velocity v along the beam propagation direction:
  - $f_\Gamma(\Delta - kv) = f_\Gamma^R + i f_\Gamma^I$

Lorentzian absorption profile

Characteristic dispersion profile

Gaussian profile

- Define $s(\Delta)$, which is directly proportional to $\chi(\Delta)$ but is independent of the specific atomic transition
- Integrate over the atomic velocity distribution to get the **Doppler-broadened** lineshape:

$$s(\Delta) = \int_{-\infty}^{+\infty} f_\Gamma(\Delta - kv) \times g_u(v) dv$$

where

$$g_u(v) = \frac{1}{\sqrt{\pi u^2}} \exp\left[-\left(\frac{v}{u}\right)^2\right]$$

is the normalized Gaussian

Δ – detuning
k – wave number
v – atomic velocity
u – thermal velocity

Convolution Theory

2 profiles due to
- Natural Broadening
- Doppler Broadening

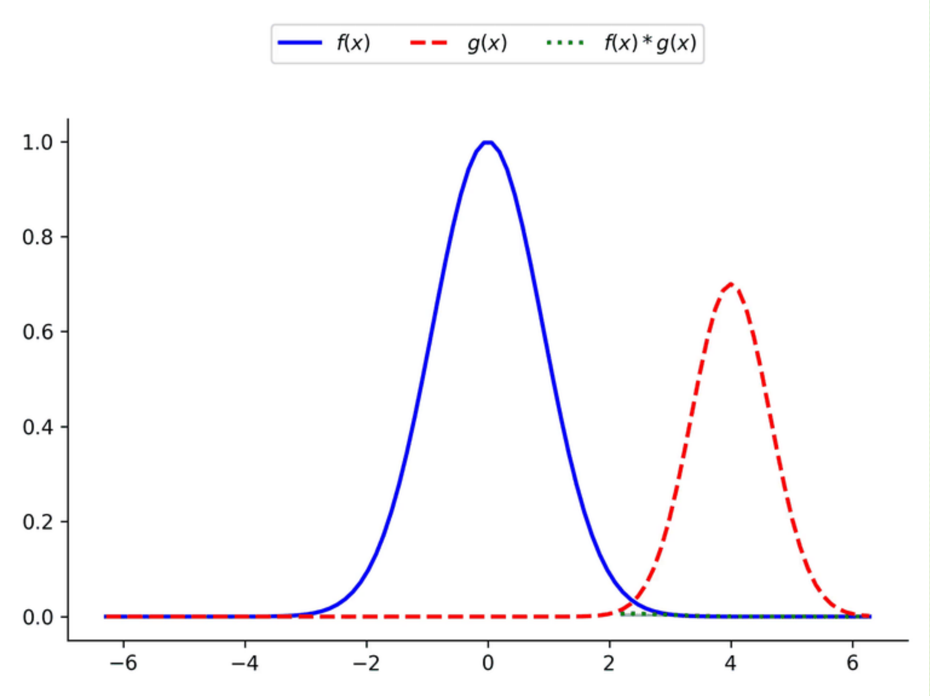
- Want to characterize the velocity profile and have an analytical expression
- $s(y) = \int_{-\infty}^{+\infty} f_a(y-x) \times g(x)dx$ $\quad y = \Delta/ku$, $x = v/u$, $a = \Gamma/ku$
  - In the form of a convolution integral, rewrite as

$$s(y) = f_a(x) \otimes g(x)$$

  - And separate the real and imaginary parts

$$s^R(y) = f_a^R(x) \otimes g(x)$$

$$s^I(y) = f_a^I(x) \otimes g(x)$$



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- Convolution trick
  1. Take the Fourier transforms of $f_a^I(x)$ and $g(x)$
     - Convolution in time domain = multiplication in frequency domain
  2. Compute the product
  3. Take the inverse Fourier transform
  4. Result: analytical expression to use in code and that gives the famous Voigt profile

Getting Back to the Principal Equation

- $s^I(y) = \frac{\sqrt{\pi}}{2} e^{\frac{1}{4}(a-i2y)^2} \left(\mathrm{Erfc}\left[\frac{a}{2} - iy\right] + e^{i2ay}\mathrm{Erfc}\left[\frac{a}{2} + iy\right]\right)$
  - Voigt profile
  - Related to absorption coefficient

$y = \Delta/ku$
$x = v/u$
$a = \Gamma/ku$

- $s^R(y) = i\frac{\sqrt{\pi}}{2} \left(e^{\frac{1}{4}(a-i2y)^2}\mathrm{Erfc}\left[\frac{a}{2} - iy\right] - e^{\frac{1}{4}(a+i2y)^2}\mathrm{Erfc}\left[\frac{a}{2} + iy\right]\right)$
  - Related to refractive index

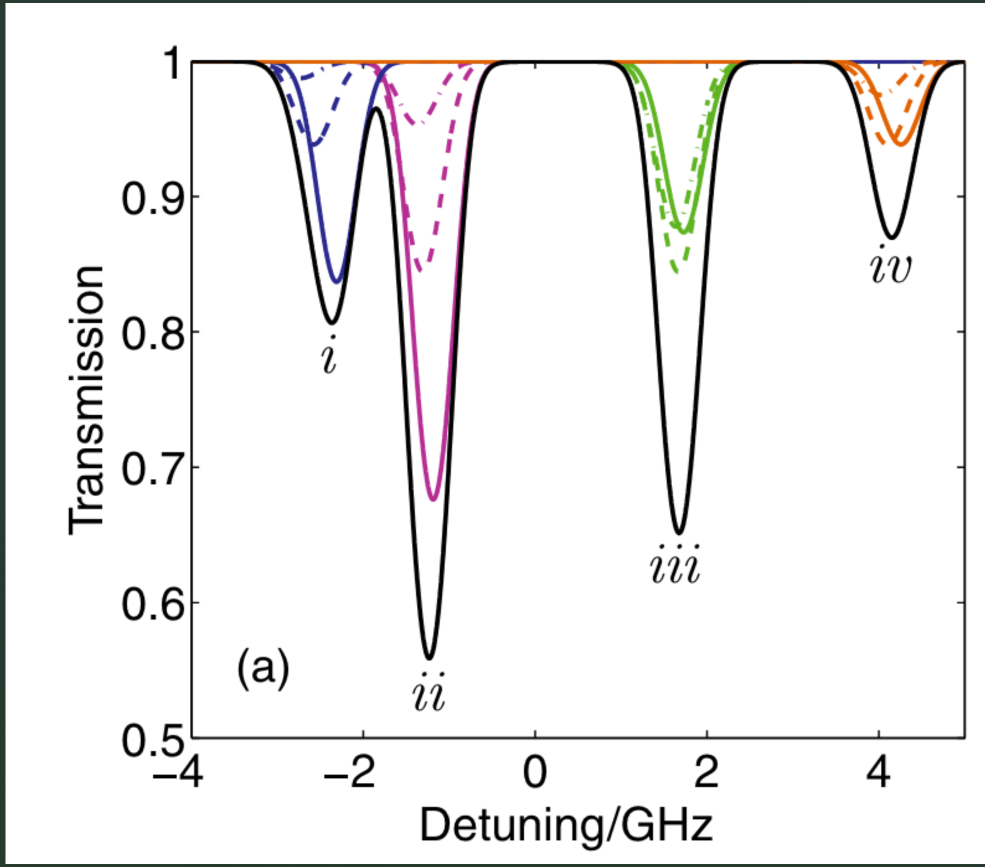
Connecting Everything Together

- End goal: Plot Transmission
  - $\mathcal{T} = \exp[-(\alpha L)]$
- We've been looking for an analytical expression for α
- α is related to χ by
  - $\alpha(\Delta) = k \,\mathrm{Im}[\chi(\Delta)]$
- χ is related to transition strengths by
  - $\mathrm{Im}[\chi(\Delta)] = C_F^2 d^2 \mathcal{N} \frac{1}{\hbar\epsilon_0} \frac{1}{\mathrm{deg}} \frac{s^I(y)}{ku}$
- Just found analytical expression for $s^I(y)$
- Now have all tools necessary to evaluate $\mathcal{T} = \exp[-(\alpha L)]$

Δ – detuning
k – wave number
v – atomic velocity
u – thermal velocity
$C_F^2 d^2$ – strength of hyperfine transition
$\mathcal{N}$ – number density
deg – degeneracy of ground state

What the Results Will Look Like

- Black line: total transmission through the cell
- Solid lines: transitions between hyperfine states $F_g \rightarrow F_e = F_g + 1$
- Dashed: $F_g \rightarrow F_e = F_g$
- Dot-dash: $F_g \rightarrow F_e = F_g - 1$
- Blue lines: $F_g = 2 \rightarrow F_e$ for 87Rb
- Orange: $F_g = 1 \rightarrow F_e$ for 87Rb
- Magenta and green: for 85Rb



Acknowledgements

- Dr. Emine Altuntas
- https://iopscience.iop.org/article/10.1088/0953-4075/41/15/155004/pdf
- https://steck.us/alkalidata/rubidium87numbers.1.6.pdf

Questions?

Quantum Number	Symbol	Possible Values	Values for $5^2S_{1/2} \rightarrow 5^2P_{3/2}$
Principal QN	n	1, 2, 3, 4, …	5
Orbital Angular Momentum QN	**L**	0 (S), 1 (P), 2 (D), 3 (F)	0 → 1
Spin Angular Momentum QN	**S**	$\pm$½	½
Total Electron Angular Momentum	**J**	$\lvert L - S\rvert \leq J \leq L + S$	½ → $^3/_2$
Total Nuclear Angular Momentum	**I**	$^3/_2$ for ^{87}Rb	$^3/_2$
Total Atomic Angular Momentum QN	**F**	$\lvert J - I\rvert \leq F \leq J + 1$	1, 2 → 0, 1, 2, 3
Magnetic QN	m_F	$-F, \dots, 0, \dots, F$	$-1, 0, 1$ $-2, -1, 0, 1, 2$ …