Notes on Operational Amplifiers (Op Amps).

Comments. The name *Op Amp* comes from "*op*erational *amp*lifier."

Op Amp Golden Rules (memorize these rules)

1) The op amp has infinite open-loop gain.
2) The input impedance of the +/− inputs is infinite. (The inputs are ideal voltmeters). The output impedance is zero. (The output is an ideal voltage source.)
3) No current flows into the +/− inputs of the op amp. This is really a restatement of golden rule **2**.
4) In a circuit with negative feedback, the output of the op amp will try to adjust its output so that the voltage difference between the + and − inputs is zero ($V_+ = V_-$).

IDEAL OP AMP BEHAVOIR.

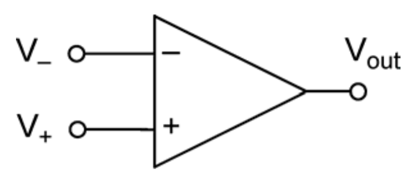
The relationship between the input ant the output of an ideal op amp (assumptions: infinite open loop gain, unlimited voltage).

$$\text{for } V_+ - V_- > 0: \quad V_{out} \to +\infty$$

$$\text{for } V_+ - V_- < 0: \quad V_{out} \to -\infty$$

$$\text{for } V_+ - V_- = 0: \quad V_{out} = 0$$

Op Amp Schematic Symbol (The upper input is usually the inverting input. Occasionally it is drawn with the non-inverting input on top when it makes the schematic easier to read. The position of the inputs may vary within the same schematic, so always look closely at the schematic!)



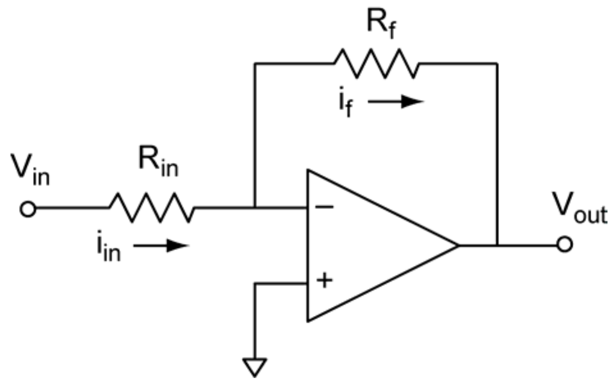
Negative Feedback. Most of the basic op amp building blocks rely on negative feedback. You can easily identify the type of feedback used by the op amp circuit. For negative feedback, the output is connected to the inverting input (− input). For positive feedback, the output is connected to the non-inverting input (+ input).

The Input Impedance of the Circuit is defined as the rate of change of V_{in} with respect to a change of I_{in}. This is simply the derivative $\frac{\mathrm{d}V_{in}}{\mathrm{d}I_{in}}$. The input impedance of the *circuit* is not in general the same as the impedance of the op amps inputs.

The Output Impedance of the Circuit, for the examples shown here, is the output impedance of the op amp. Output impedance is defined as the rate of change of V_{out} with respect to a change of I_{out}. This is simply the derivative $\frac{\mathrm{d}V_{out}}{\mathrm{d}I_{out}}$. For the ideal op amp, the output impedance is zero.

Basic Op Amp Building Blocks

Inverting Amplifier



$$V_{out} = -\frac{R_f}{R_{in}} V_{in}$$

$$A_V = \frac{V_{out}}{V_{in}} = -\frac{R_f}{R_{in}}$$

$$\frac{\mathrm{d}V_{in}}{\mathrm{d}I_{in}} = R_{in}$$

Analysis of the inverting amplifier starts with our op amp golden rules. From rule #4 we know that $V_- = V_+$ and that $V_- = 0$ because V_+ is connected to ground. From rule #3 we know that $I_{in} = I_f$ because no current flows into the inverting input.

$$V_- = V_+ \qquad V_+ = 0 \qquad I_{in} = I_f = I$$

Then we can find the relationship between V_{in} and V_{out} using Ohm's law (OL) and Kirchhoff's voltage law (KVL).

$$V_{in} - V_- = I_{in} R_{in} \qquad V_{in} - 0 = IR_{in} \qquad V_{in} = IR_{in} \qquad I = \frac{V_{in}}{R_{in}}$$

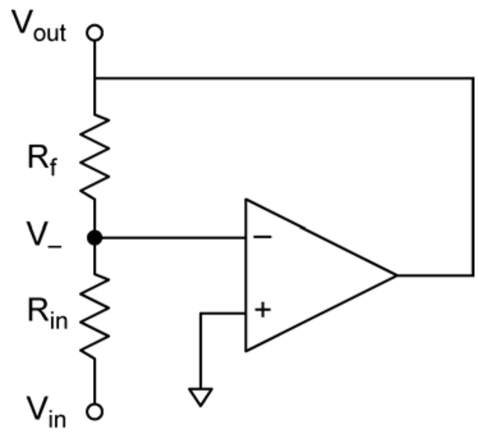
$$V_- - V_{out} = I_f R_f \qquad 0 - V_{out} = IR_f \qquad V_{out} = -IR_f$$

$$V_{out} = -\frac{R_f}{R_{in}} V_{in}$$

The voltage gain A_V is the derivative of V_{out} with respect to V_{in}. When the amplifier has only one input and $V_{out} = 0$ when $V_{in} = 0$, we will make the assumption that $A_V = V_{out}/V_{in}$.

$$A_v = \frac{V_{out}}{V_{in}} = \frac{-IR_f}{IR_{in}} = -\frac{R_f}{R_{in}}$$

Alternatively we could have started our analysis from the voltage divider formed by R_f and R_{in}. The voltage divider will relate the voltage at V_- with V_{out} and V_{in}. In this case The total voltage across the divider is $V_{out} - V_{in}$. Because the bottom end of the divider is not connected to ground, we must add the extra V_{in} term to offset V_-. We arrive at the same result.



$$V_- = (V_{out} - V_{in})\left(\frac{R_{in}}{R_{in}+R_f}\right) + V_{in} = 0$$

$$V_{out}\left(\frac{R_{in}}{R_{in}+R_f}\right) = V_{in}\left(\frac{R_{in}}{R_{in}+R_f}\right) - V_{in}$$

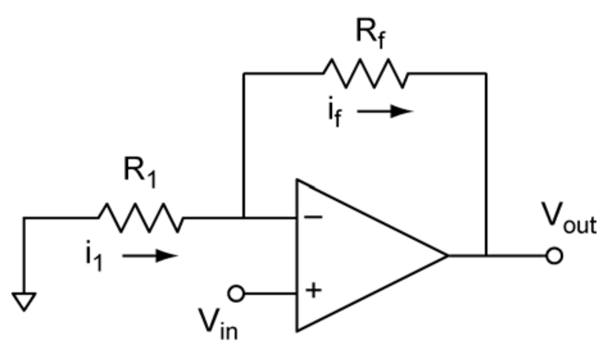
$$V_{out}\left(\frac{R_{in}}{R_{in}+R_f}\right)\left(\frac{R_{in}+R_f}{R_{in}}\right) = V_{in}\left(\frac{R_{in}}{R_{in}+R_f}\right)\left(\frac{R_{in}+R_f}{R_{in}}\right) - V_{in}\left(\frac{R_{in}+R_f}{R_{in}}\right)$$

$$V_{out} = V_{in} - V_{in}\left(\frac{R_{in}+R_f}{R_{in}}\right) = V_{in}\left(1 - \frac{R_{in}+R_f}{R_{in}}\right) = -\frac{R_f}{R_{in}}V_{in}$$

$$V_{out} = -\frac{R_f}{R_{in}}V_{in}$$

The input impedance of the inverting amplifier is determined by R_{in}. Note that V_- is held at the same voltage as V_+ by the op amp feedback. Because V_+ is connected to ground, the input impedance is just R_{in}.

Non-inverting Amplifier



$$V_{out} = \left(1 + \frac{R_f}{R_{in}}\right)V_{in}$$

$$A_V = \frac{V_{out}}{V_{in}} = 1 + \frac{R_f}{R_{in}}$$

$$\frac{\mathrm{d}V_{in}}{\mathrm{d}I_{in}} \to \infty$$

Analysis of the non-inverting amplifier starts with our op amp golden rules. From rule #4 we know that $V_- = V_+$ and that $V_- = V_{in}$ because V_+ is connected to V_{in}. From rule #3 we know that $I_{in} = I_f$ because no current flows into the inverting input.

$$V_- = V_+ \qquad V_+ = V_{in} \qquad I_{in} = I_f = I$$

Then we can find the relationship between V_{in} and V_{out} using Ohm's law (OL) and Kirchhoff's voltage law (KVL).

$$-V_- = I_1 R_1 \qquad -V_{in} = IR_1 \qquad I = -\frac{V_{in}}{R_1}$$

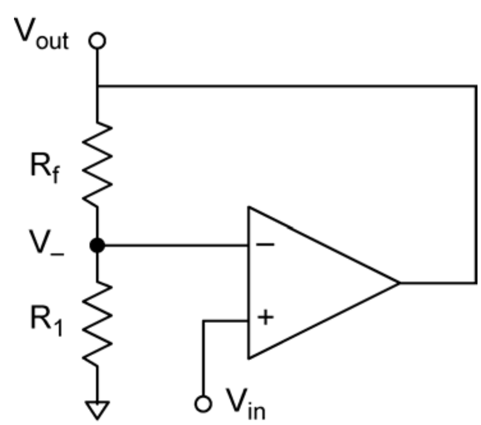
$$V_- - V_{out} = I_f R_f \qquad V_{in} - V_{out} = IR_f \qquad V_{out} = V_{in} - IR_f$$

$$V_{out} = V_{in} - \left(-\frac{V_{in}}{R_1}\right) R_f = V_{in}\left(1 + \frac{R_f}{R_{in}}\right)$$

The voltage gain A_V is the derivative of V_{out} with respect to V_{in}. When the amplifier has only one input and $V_{out} = 0$ when $V_{in} = 0$, we will make the assumption that $A_V = V_{out}/V_{in}$.

$$A_v = \frac{V_{out}}{V_{in}} = \left(1 + \frac{R_f}{R_{in}}\right)$$

Alternatively we could have started out analysis from the voltage divider formed by R_f and R_{in}. The voltage divider will relate the voltage at V_- with V_{out} and V_{in}. In this case The total voltage across the divider is V_{out} and the we know that $V_- = V_{in}$. We arrive at the same result.



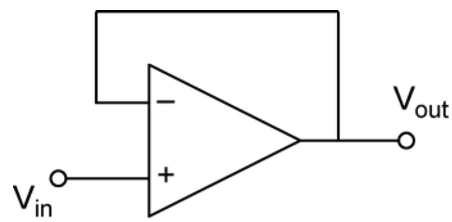
$$V_- = V_{out}\left(\frac{R_1}{R_1 + R_f}\right) = V_{in}$$

$$V_{out}\left(\frac{R_1}{R_1 + R_f}\right)\left(\frac{R_1 + R_f}{R_1}\right) = V_{in}\left(\frac{R_1 + R_f}{R_1}\right) = V_{in}\left(1 + \frac{R_f}{R_1}\right)$$

$$V_{out} = V_{in}\left(1 + \frac{R_f}{R_1}\right)$$

The input impedance of the follower is the input impedance of the op amps input. For an ideal op amp the input impedance is infinite.

Voltage Follower



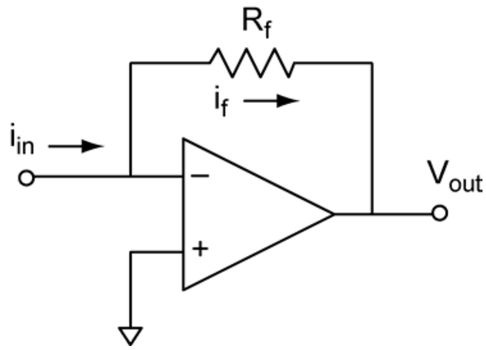
$$V_{out} = V_{in}$$

$$A_V = \frac{V_{out}}{V_{in}} = 1$$

$$\frac{\mathrm{d}V_{in}}{\mathrm{d}I_{in}} \to \infty$$

This is a special case of the non-inverting amplifier with $R_{\mathrm{in}} \to \infty$ and $R_{\mathrm{f}} = 0$. The follower has a very high input impedance. Voltage follower has application when the source voltage can not supply very much current, a pH meter for example.

Current-to-Voltage Converter (AKA, I-V Converter, Transimpedance Amplifier). This circuit takes an input current and converts it to an output voltage. The input impedance of the ideal current to voltage converter is zero (the ideal current meter).



$$V_{out} = -R_f I_{in}$$

$$A_Z = \frac{V_{out}}{I_{in}} = -R_f$$

$$\frac{\mathrm{d}V_{in}}{\mathrm{d}I_{in}} = 0$$

Analysis of the current-to-voltage converter starts with our op amp golden rules. From rule #4 we know that $V_- = V_+$ and that $V_- = 0$ because V_+ is connected to ground. From rule #3 we know that $I_{in} = I_f$ because no current flows into the inverting input.

$$V_- = V_+ \qquad V_+ = 0 \qquad I_{in} = I_f = I$$

Then we can find the relationship between V_{in} and V_{out} using Ohm's law (OL) and Kirchhoff's voltage law (KVL).

$$I_{in} = I_f$$

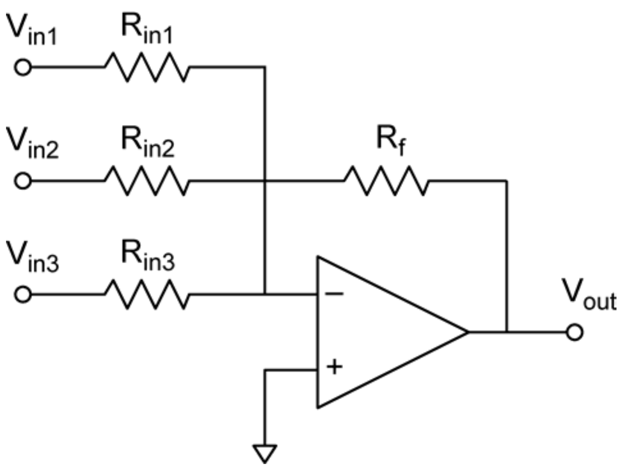
$$V_- - V_{out} = I_f R_f \qquad 0 - V_{out} = I_f R_f \qquad V_{out} = -I_f R_f$$

$$V_{out} = -I_f R_f = -I_{in} R_f$$

The current-to-voltage converter has transimpedance gain. Transimpedance gain is not unitless, it has units of impedance (Ohms). The transimpedance gain A_{Z} is the derivative of V_{out} with respect to I_{in}. When the amplifier has only one input and V_{out} = 0 when I_{in} = 0, we will make the assumption that $A_V = V_{out}/I_{in}$.

$$A_Z = \frac{V_{out}}{I_{in}} = \frac{-I_{in}R_f}{I_{in}} = -R_f$$

Summing Amplifier. This circuit will add (and subtract) the input voltages. Subtraction is accomplished by inverting the voltages before adding them. Note that summing can only occur for inputs to the inverting side of the op amp. This is because of the V_- node is a current summing junction where the input currents sum to the feedback current.

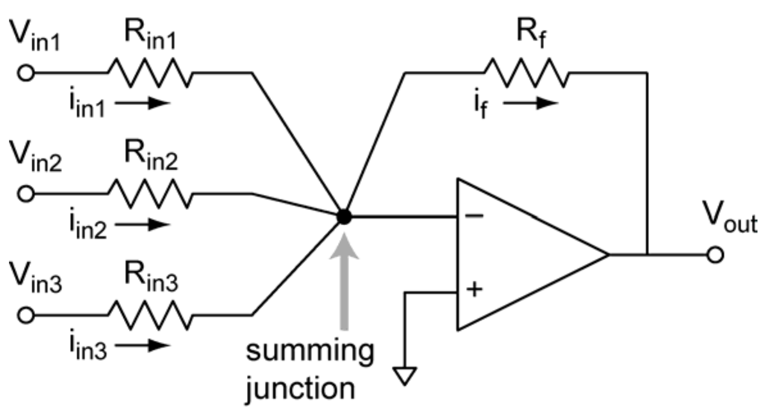


$$V_{out} = A_{V1}V_{in1} + A_{V2}V_{in2} + A_{V3}V_{in3} + \cdots$$

$$\text{the gain of the } n\text{th input: } A_{Vn} = \frac{\mathrm{d}V_{out}}{\mathrm{d}V_{inn}} = -\frac{R_f}{R_{inn}}$$

$$\text{the impedance of the } n\text{th input: } Z_{inn} = \frac{\mathrm{d}V_{inn}}{\mathrm{d}I_{inn}} = R_{inn}$$

This is another look at the summing amplifier that emphases the summing junction.



Analysis of the summing amplifier starts with our op amp golden rules. From rule #4 we know that $V_- = V_+$ and that $V_- = 0$ because V_+ is connected to ground. From rule #3 we know that $\sum I_{inn} = I_f$ because no current flows into the inverting input. (I_{inn} is the current of the nth input.)

$$V_- = V_+ \qquad V_+ = 0 \qquad \sum I_{inn} = I_{in1} + I_{in2} + I_{in3} = I_f$$

Then we can find the relationship between V_{in} and V_{out} using Ohm's law (OL) and Kirchhoff's voltage law (KVL).

$$V_{inn} - V_- = I_{inn} R_{inn} \qquad V_{inn} - 0 = IR_{inn} \qquad V_{inn} = I_{inn} R_{inn} \qquad I_{inn} = \frac{V_{inn}}{R_{inn}}$$

$$V_- - V_{out} = I_f R_f \qquad 0 - V_{out} = IR_f \qquad V_{out} = -I_f R_f$$

$$V_{out} = -I_f R_f = -R_f \left(I_{in1} + I_{in2} + I_{in3} \right) = -R_f \left(\frac{V_{in1}}{R_{in1}} + \frac{V_{in2}}{R_{in2}} + \frac{V_{in3}}{R_{in3}} \right) = -\frac{R_f}{R_{in1}} V_{in1} - \frac{R_f}{R_{in2}} V_{in2} - \frac{R_f}{R_{in3}} V_{in3}$$

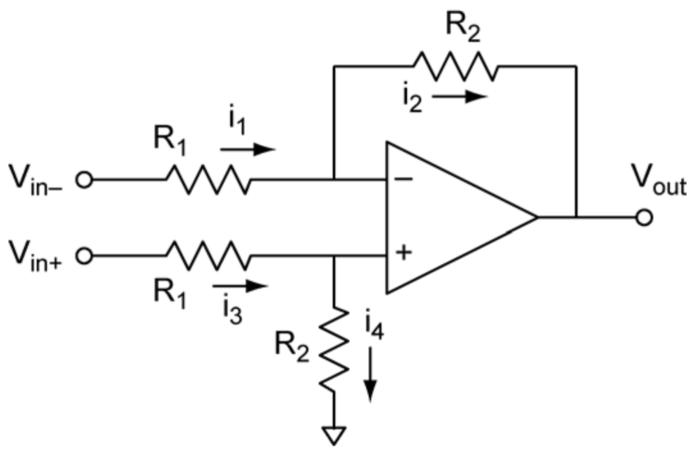
$$V_{out} = -\frac{R_f}{R_{in1}} V_{in1} - \frac{R_f}{R_{in2}} V_{in2} - \frac{R_f}{R_{in3}} V_{in3}$$

The voltage gain A_V is the derivative of V_{out} with respect to V_{in}.

$$V_{out} = A_{V1} V_{in1} + A_{V2} V_{in2} + A_{V3} V_{in3} + \cdots$$

$$\text{the gain of the } n\text{th input}: \; A_{Vn} = \frac{\mathrm{d} V_{out}}{\mathrm{d} V_{inn}} = -\frac{R_f}{R_{inn}}$$

Differential Amplifier. The term *differential* is used in the sense of *difference*. Do not confuse the differential amplifier with the differentiator. One important application of the differential amplifier over comes the problem of grounding that you encountered in lab when using the oscilloscope to make measurements. The typical oscilloscope always performs voltage measurements with respect to is own ground. A differential amplifier used before the scope input could measure the $V_{+\mathrm{in}}$ with respect to $V_{-\mathrm{in}}$. The ground of the differential amplifier would be connected to the ground of the scope for this application, so the V_{out} will be measured correctly.



$$V_{out} = \frac{R_2}{R_1} \left(V_{+in} - V_{-in} \right)$$

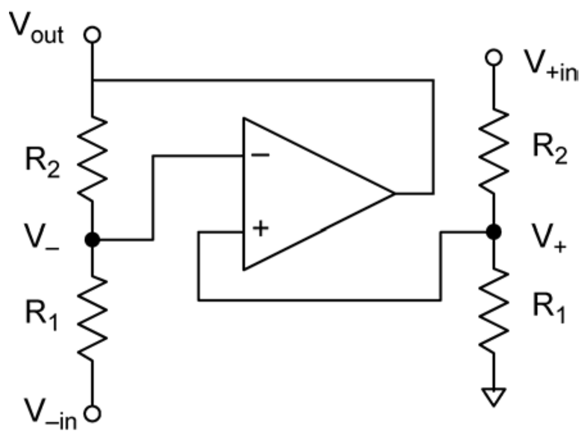
$$\text{the gain for } V_{+\mathrm{in}}: \; A_{V+in} = \frac{\mathrm{d} V_{out}}{\mathrm{d} V_{+in}} = +\frac{R_2}{R_1}$$

$$\text{the gain for } V_{-\mathrm{in}}: \; A_{V-in} = \frac{\mathrm{d} V_{out}}{\mathrm{d} V_{-in}} = -\frac{R_2}{R_1}$$

Analysis of the differential amplifier starts with our op amp golden rules. From rule #4 we know that $V_- = V_+$. From rule #3 we know that $I_1 = I_2$ and that $I_3 = I_4$ because no current flows into the inputs.

$$V_- = V_+ \qquad I_1 = I_2 \qquad I_3 = I_4$$

Then we can find the relationship between $V_{+\text{in}}$, $V_{-\text{in}}$, and V_{out} using the voltage divider equations. We recognize that $V_- = V_+$ and that V_+ will be the output of the voltage divider formed by the two resistors connected to the non-inverting input. The voltage at V_- is the output of the voltage divider formed by the two resistors connected to the inverting input.



$$V_- = \left(V_{out} - V_{-in}\right)\left(\frac{R_1}{R_1 + R_2}\right) + V_{-in} \qquad V_+ = V_{+in}\left(\frac{R_2}{R_1 + R_2}\right)$$

$$V_- = V_+$$

$$\left(V_{out} - V_{-in}\right)\left(\frac{R_1}{R_1 + R_2}\right) + V_{-in} = V_{+in}\left(\frac{R_2}{R_1 + R_2}\right)$$

$$V_{out}\left(\frac{R_1}{R_1 + R_2}\right) - V_{-in}\left(\frac{R_1}{R_1 + R_2}\right) + V_{-in} = V_{+in}\left(\frac{R_2}{R_1 + R_2}\right)$$

$$V_{out}\left(\frac{R_1}{R_1 + R_2}\right) = V_{+in}\left(\frac{R_2}{R_1 + R_2}\right) + V_{-in}\left(\frac{R_1}{R_1 + R_2}\right) - V_{-in}$$

$$V_{out}\left(\frac{R_1}{R_1 + R_2}\right)\left(\frac{R_1 + R_2}{R_1}\right) = V_{+in}\left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{R_1 + R_2}{R_1}\right) + V_{-in}\left(\frac{R_1}{R_1 + R_2}\right)\left(\frac{R_1 + R_2}{R_1}\right) - V_{-in}\left(\frac{R_1 + R_2}{R_1}\right)$$

$$V_{out} = \left(\frac{R_2}{R_1}\right)V_{+in} - \left(\frac{R_2}{R_1}\right)V_{-in}$$

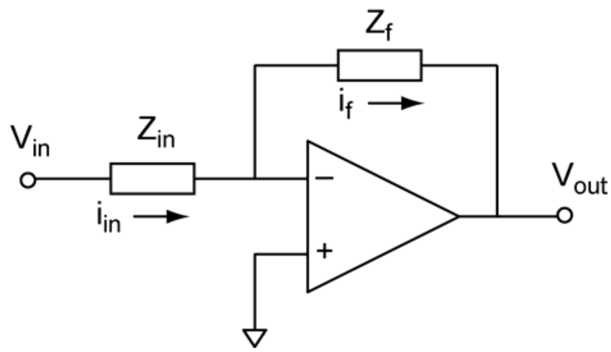
$$V_{out} = \frac{R_f}{R_{in}}\left(V_{+in} - V_{-in}\right)$$

The voltage gain A_V is the derivative of V_{out} with respect to each input V_{in}.

$$\text{the gain for } V_{+\text{in}}: \ A_{V+in} = \frac{\mathrm{d}V_{out}}{\mathrm{d}V_{+in}} = +\frac{R_2}{R_1}$$

$$\text{the gain for } V_{-\text{in}}: \ A_{V-in} = \frac{\mathrm{d}V_{out}}{\mathrm{d}V_{-in}} = -\frac{R_2}{R_1}$$

The inverting amplifier with generalized impedances. The results derived above can be extended to general impedances. Note that Z_f and Z_{in} can be the impedance of any network. The following are examples of the inverting amplifier, but ANY of the previous examples can be generalized in this way.

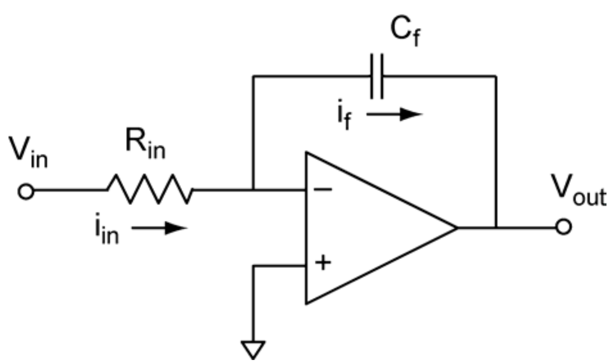


$$V_{out} = -\frac{Z_f}{Z_{in}} V_{in}$$

$$A_V = \frac{V_{out}}{V_{in}} = -\frac{Z_f}{Z_{in}}$$

$$\frac{\mathrm{d}V_{in}}{\mathrm{d}I_{in}} = Z_{in}$$

Integrator. A capacitor as the feedback impedance.



$$V_{out}(\omega) = \frac{j}{\omega R_{in} C_f} V_{in}(\omega)$$

$$\left|V_{out}(\omega)\right| = \frac{1}{\omega R_{in} C_f} V_{in}(\omega)$$

$$V_{out}(t) = -\frac{1}{R_{in} C_f} \int V_{in}(t)\,\mathrm{d}t$$

$$A_V(\omega) = \frac{V_{out}}{V_{in}} = \frac{j}{\omega R_{in} C_f}$$

$$\left|A_V(\omega)\right| = \left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\omega R_{in} C_f}$$

$$\frac{\mathrm{d}V_{in}}{\mathrm{d}I_{in}} = R_{in}$$

Analysis of the integrator in the frequency domain is a simple extension of our generalized result for the inverting amplifier.

$$Z_{in} = R_{in} \qquad Z_f = \frac{1}{j\omega C_f} \qquad V_{out} = -\frac{Z_f}{Z_{in}}$$

$$V_{out} = -\frac{Z_f}{Z_{in}} V_{in} = -\frac{\frac{1}{j\omega C_f}}{R_{in}} V_{in} = -\frac{1}{j\omega R_{in} C_f} V_{in} = \frac{j}{\omega R_{in} C_f} V_{in}$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{j}{\omega R_{in} C_f}$$

$$|A_V| = \left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\omega R_{in} C_f}$$

Time Domain Analysis of the Integrator starts with our op amp golden rules. From rule #4 we know that $V_- = V_+$ and that $V_- = 0$ because V_+ is connected to ground. From rule #3 we know that $I_{in} = I_f$ because no current flows into the inverting input.

$$V_- = V_+ \qquad V_+ = 0 \qquad I_{in} = I_f = I$$

Then we can find the relationship between V_{in} and V_{out} using Ohm's law (OL) and Kirchhoff's voltage law (KVL).

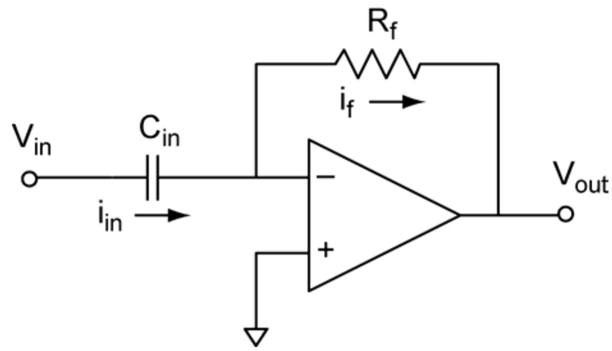
$$V_{in} - V_- = I_{in} R_{in} \qquad V_{in} - 0 = I R_{in} \qquad V_{in} = I R_{in} \qquad I = \frac{V_{in}}{R_{in}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(V_- - V_{out}\right) = \frac{I_f}{C_f} \qquad \frac{\mathrm{d}}{\mathrm{d}t}\left(0 - V_{out}\right) = \frac{I}{C_f} \qquad \frac{\mathrm{d}}{\mathrm{d}t} V_{out} = -\frac{I}{C_f} \qquad V_{out} = -\frac{1}{C_f}\int I \, \mathrm{d}t$$

$$V_{out} = -\frac{1}{C_f}\int I \, \mathrm{d}t = -\frac{1}{C_f}\int \frac{V_{in}}{R_{in}} \mathrm{d}t = -\frac{1}{R_{in} C_f}\int V_{in} \, \mathrm{d}t$$

$$V_{out} = -\frac{1}{R_{in} C_f}\int V_{in} \, \mathrm{d}t$$

Differentiator. A capacitor as the input impedance.



$$V_{out}(\omega) = -j\omega R_f C_{in} V_{in}(\omega)$$

$$\left|V_{out}(\omega)\right| = -\omega R_f C_{in} V_{in}(\omega)$$

$$V_{out}(t) = -R_f C_{in} \frac{\mathrm{d}V_{in}(t)}{\mathrm{d}t}$$

$$A_V(\omega) = \frac{V_{out}}{V_{in}} = -j\omega R_f C_{in}$$

$$\left|A_V(\omega)\right| = \left|\frac{V_{out}}{V_{in}}\right| = -\omega R_f C_{in}$$

$$\frac{\mathrm{d}V_{in}}{\mathrm{d}I_{in}} = \frac{1}{j\omega C_{in}}$$

Analysis of the differentiator in the frequency domain is a simple extension of our generalized result for the inverting amplifier.

$$Z_{in} = \frac{1}{j\omega C_{in}} \qquad Z_f = R_f \qquad V_{out} = -\frac{Z_f}{Z_{in}}$$

$$V_{out} = -\frac{Z_f}{Z_{in}} V_{in} = -\frac{R_f}{\dfrac{1}{j\omega C_{in}}} V_{in} = -j\omega R_f C_{in} V_{in}$$

$$\left|V_{out}\right| = \omega R_f C_{in} V_{in}$$

$$A_V = \frac{V_{out}}{V_{in}} = -j\omega R_f C_{in}$$

$$\left|A_V\right| = \left|\frac{V_{out}}{V_{in}}\right| = \omega R_f C_{in}$$

Time Domain Analysis of the Differentiator starts with our op amp golden rules. From rule #4 we know that $V_- = V_+$ and that $V_- = 0$ because V_+ is connected to ground. From rule #3 we know that $I_{in} = I_f$ because no current flows into the inverting input.

$$V_- = V_+ \qquad V_+ = 0 \qquad I_{in} = I_f = I$$

Then we can find the relationship between V_{in} and V_{out} using Ohm's law (OL) and Kirchhoff's voltage law (KVL).

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(V_{in} - V_{-}\right) = \frac{I_{in}}{C_{in}} \qquad \frac{\mathrm{d}}{\mathrm{d}t}V_{in} = \frac{I}{C_{in}} \qquad C_{in}\frac{\mathrm{d}}{\mathrm{d}t}V_{in} = I$$

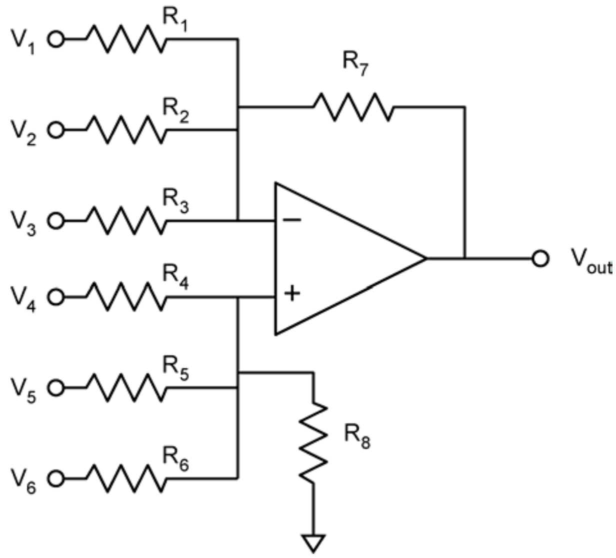
$$V_{-} - V_{out} = I_f R_f \qquad 0 - V_{out} = IR_f \qquad V_{out} = -IR_f$$

$$V_{out} = -\left(C_{in}\frac{\mathrm{d}}{\mathrm{d}t}V_{out}\right)R_f = -R_f C_{in}\frac{\mathrm{d}}{\mathrm{d}t}V_{in}$$

$$V_{out} = -R_f C_{in}\frac{\mathrm{d}}{\mathrm{d}t}V_{in}$$

The General Op Amp Circuit

Example: an op amp circuit with 3 inverting and 3 non-inverting inputs



What can we say about such a complicated looking amplifier? Don't panic, we can use what we have learned from the above analyses to painlessly arrive at the solution. Without doing any analysis, what can we say? First, we know that

$$V_{out} = A_{V1}V_1 + A_{V2}V_2 + A_{V3}V_3 + A_{V4}V_4 + A_{V5}V_5 + A_{V6}V_6$$

$$\text{the gain of the } n\text{th input}: \; A_{Vn} = \frac{\mathrm{d}V_{out}}{\mathrm{d}V_n}$$

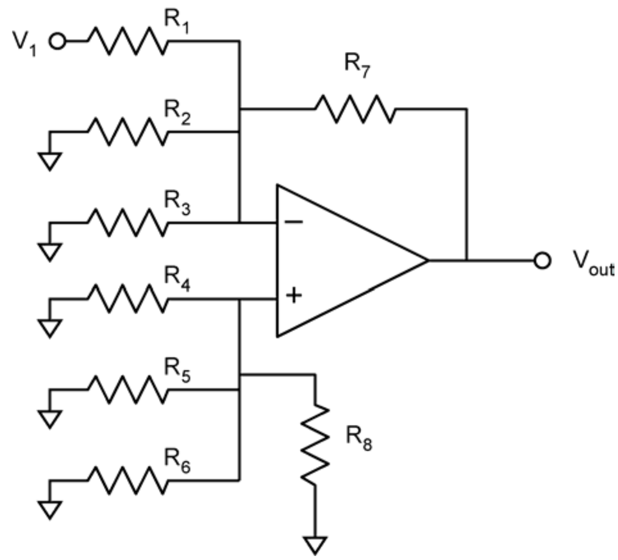
Second, we know that the voltage gains for V_1, V_2, and V_3 will be inverting (negative) and that the voltage gains for V_4, V_5, and V_6 will be non-inverting (positive). We could simply blaze away at the problem by applying the op amp golden rules just like we did for the derivations for the basic op amp building blocks, but there is a better way.

The Strategy.

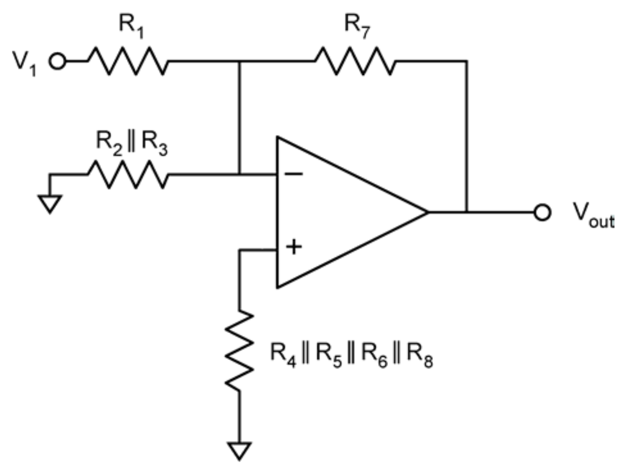
We will make use of the results for the basic op amp building blocks and the principle of superposition (e.g. the inverting amplifier and the non-inverting amplifier). To apply the principle of super position, we analyze the gain for one input at a time and *turn off* all the other inputs (set them to zero). We treat each input as an ideal voltage source, so that **an input that is *turned off* is equivalent to a connecting the input terminal directly to ground.** The inverting inputs and the non-inverting inputs will behave differently.

Analysis of the Inverting Inputs by Superposition

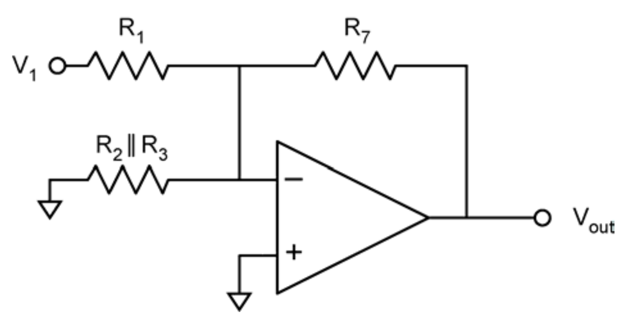
Let's first use analyze the voltage gain A_{V1} of the input V_1. We proceed by connecting all the other inputs to ground. (Analysis of the voltage gain for the other inverting inputs (A_{V2} and A_{V3}) is analogous.)



This simplifies to:



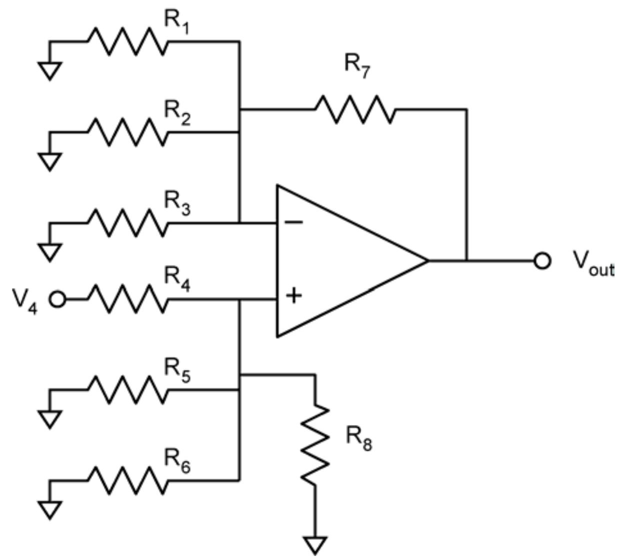
Notice that the non-inverting input, V_+ is connected to ground though a resistor. Because no current flows into the op amp's inputs, $V_+ = 0$, equivalent to it being connected directly to ground (see below).



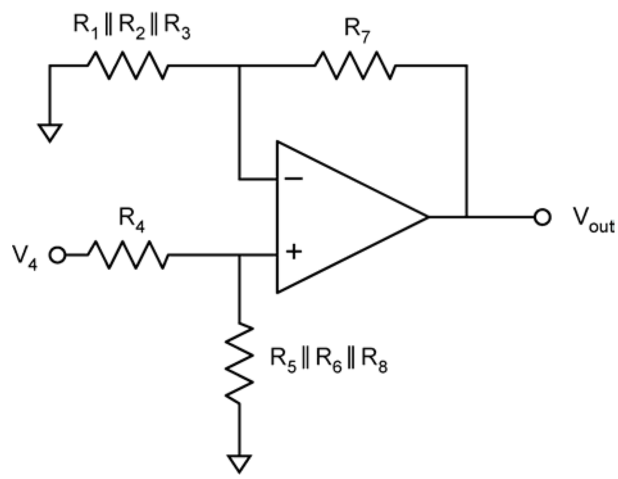
This looks like a summing amplifier with $V_2 = V_3 = 0$. The result for the summing amplifier is $A_{V1} = -\dfrac{R_7}{R_1}$. We can do the same analysis for each inverting input. (Why don't the two resistors R_2 and R_3 enter this analysis for A_{V1}? Hint: Consider the voltage across these two resistors.)

Analysis of the Non-Inverting Inputs by Superposition

Now let's use analyze the voltage gain A_{V4} of the non-inverting input V_4. We proceed by connecting all the other inputs to ground. (Analysis of the voltage gain for the other non-inverting inputs (A_{V5} and A_{V6}) is analogous.)



This simplifies to:



This is a non-inverting amplifier. The resistors on the non-inverting side, R_4, R_5, R_6, and R_8, form a voltage divider that reduces the voltage seen by V_+. It is the voltage at V_+ that is seen by the op amp. The voltage gain of V_+ is determined by the resistors on the inverting side, R_1, R_2, R_3, and R_7. Hence the voltage gain A_{V4} of the V_4 input has a term from the voltage divider, relating V_4 to V_+, and a term for the voltage gain of V_+, relating V_+ to V_{out}. The voltage gain A_{V4} is the product of the two term and relates V_4 to V_{out}.

$$A_{V4} = \left(\frac{R_5 \| R_6 \| R_7}{R_4 + R_5 \| R_6 \| R_7} \right) \left(1 + \frac{R_7}{R_1 \| R_2 \| R_3} \right)$$

The voltage gains for the other non-inverting inputs can be found in this way.

An important observation is that multiple inputs into the non-inverting side of the op amp do not sum in the simple way that they do for inverting inputs. Thus the summing amplifier that we listed as a basic building block does not have a non-inverting analog! (If we need a non-inverted sum, we just follow the summing amplifier with a unity gain inverting amplifier.)