Physics 2414
Group Exercise 10

Name 1: ________ OUID 1: ________

Name 2: ________ OUID 2: ________

Name 3: ________ OUID 3: ________

Name 4: ________ OUID 4: ________

Solutions Solutions

Section Number: ____

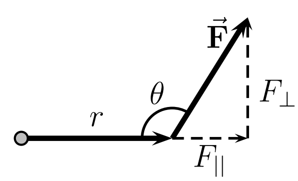
Torque

Torque: Torque about an axis due to a force $\vec{\mathbf{F}}$ is determined by the expression

$$\tau = rF_{\perp} = r|\vec{\mathbf{F}}||\sin\theta| \tag{1}$$

where r is the distance between the axis and the point of action of the force, and $F_{\perp}$ is the component of the force that contributes to the rotation. In the diagram shown

$$F_{\perp} = |\vec{\mathbf{F}}||\sin\theta|. \tag{2}$$



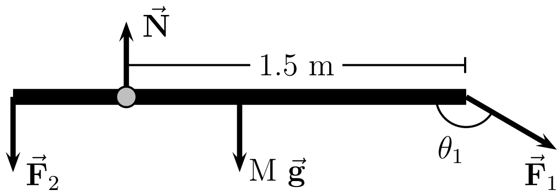
Newton's second law for rotation of a rigid body with rotational inertia I about an axis is

$$\tau_{\text{net}} = I\alpha \tag{3}$$

where τ_{net} is the sum of all the torques acting on the rigid body, and α is the angular acceleration of the rigid body about the axis.

Problems

Consider a uniform thin rod of mass $M = 2$ kg and length $L = 2$ meters nailed to the wall such that the rod is free to rotate about the nail. The nail is 0.5 meters from one end of the rod. Assume no friction between the nail and rod. The force of gravity $M\vec{\mathbf{g}}$ and the normal force $\vec{\mathbf{N}}$ act on the rod. Two more forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ are exerted on the rod as shown in the figure.



1. *Rotational inertia:*

(a) What is the rotational inertia I_{end} of a uniform thin rod rotating about an axis passing through one end of the rod? (Hint: refer to Figure 8-21 in the textbook.)

$$I_{\text{end}} = \tfrac{1}{3}ML^2 \tag{4}$$

Note: We shall not be using the rotatioal inertia you found in eqn. (4) above. Rotational inertia of a rigid body depends on the axis about which it is rotating. The rotational inertia of the rod about the axis in our problem is given by

$$I = \frac{7}{48}ML^2 \tag{5}$$

2. *Torque due to* $\vec{\mathbf{F}}_1$*:*

(a) What is the distance r between the axis of rotation and the point where the force $\vec{\mathbf{F}}_1$ acts.

$$r_1 = \tfrac{3}{2} \text{ meter} \tag{6}$$

(b) The force $\vec{\mathbf{F}}_1$ can be broken into a parallel ($||$) component and a perpendicular ($\perp$) component. How much does the parallel component of the force help in rotating the rod about the axis?

Zero contribution.

(c) Write the expression for the perpendicular component of $\vec{\mathbf{F}}_1$ in terms of the magnitude $|\vec{\mathbf{F}}_1|$ and the angle θ_1.

$$F_{1\perp} = |\vec{\mathbf{F}}_1||\sin\theta_1| \tag{7}$$

(d) Determine the magnitude of the torque about the axis due to the force $\vec{\mathbf{F}}_1$, if $|\vec{\mathbf{F}}_1| = 10$ Newtons and $\theta_1 = 150^o$.

$$\tau_1 = r_1|\vec{\mathbf{F}}_1||\sin\theta_1| \tag{8}$$
$$= \frac{3}{2} \times 10 \times |\sin 150| = 7.5 \text{ Newton-meter} \tag{9}$$

(e) What is the direction of the torque τ_1? (Hint: Ask the question in which direction would τ_1 alone move the rod.)

√ (i) clockwise (-ve)

(ii) anticlockwise (+ve)

3. *Torque due to* $\vec{\mathbf{F}}_2$*:*

(a) Determine the magnitude of the torque on the rod due to the force $\vec{\mathbf{F}}_2$, if $|\vec{\mathbf{F}}_2| = 5$ Newtons.

$$\tau_2 = r_2|\vec{\mathbf{F}}_2||\sin\theta_2| \tag{10}$$
$$= \frac{1}{2} \times 5 \times \sin 90 = 2.5 \text{ Newton-meter} \tag{11}$$

(b) What is the direction of the torque τ_2?

(i) clockwise (-ve)

√ (ii) anticlockwise (+ve)

4. *Torque due to gravity force* $M\vec{\mathbf{g}}$*:*

(a) At what point with respect to the axis of rotation does the gravity force act on the rod?

$$r_{Mg} = \frac{1}{2} \text{ meter} \tag{12}$$

(b) Determine the magnitude of the torque on the rod due to the force $M\vec{\mathbf{g}}$.

$$\tau_{Mg} = r_{Mg} Mg \sin\theta_g \tag{13}$$
$$= \frac{1}{2} \times 2 \times 9.8 \times \sin 90 = 9.8 \text{ Newton-meter} \tag{14}$$

(c) What is the direction of the torque τ_{Mg}?

√ (i) clockwise (-ve)

(ii) anticlockwise (+ve)

5. *Torque due to normal force* $\vec{\mathbf{N}}$*:*

(a) At what point with respect to the axis of rotation does the normal force act on the rod?

$$r_N = 0 \tag{15}$$

(b) Determine the magnitude of the torque on the rod due to the force $\vec{\mathbf{N}}$.

$$\tau_N = r_N |\vec{\mathbf{N}}| \sin 90 = 0 \tag{16}$$

(c) What is the direction of the torque τ_N?

(i) clockwise (-ve)

(ii) anticlockwise (+ve)

√ (iii) no direction

6. *Newton's law for rotation of rigid bodies:*

(a) Determine the net torque τ_{net} on the rod about the axis. (Caution: Remember to take care of the directions of individual torques.)

$$\tau_{\text{net}} = \tau_1 + \tau_2 + \tau_{Mg} + \tau_N \tag{17}$$

$$= -7.5 + 2.5 - 9.8 + 0 = -14.8 \text{ Newton-meter} \tag{18}$$

(b) Determine the angular acceleration of the rod about the axis. (Hint: $\tau_{\text{net}} = I\alpha$.)

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{\tau_{\text{net}}}{\frac{7}{48}ML^2} = \frac{-14.8}{\frac{7}{48} \times 2 \times 2^2} = -12.7 \text{ rad/sec}^2 \tag{19}$$

(c) What is the direction of the angular acceleration?

√ (i) clockwise (-ve)

(ii) anticlockwise (+ve)