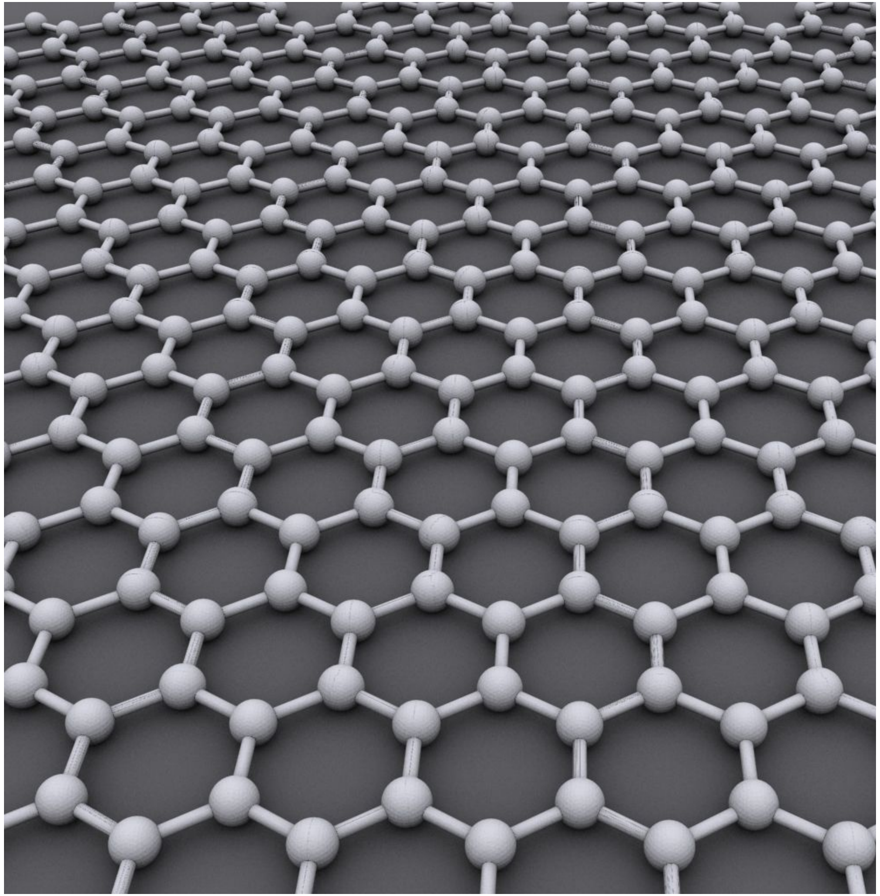
OU Physics REU

Kevin Wen; Five Minute Talk

Topological Materials and Flat Bands:

Perspectives from Math and Physics



Overview

1. What are Topological Materials?
2. Perspectives from Math
3. Perspectives from Physics
4. What are Flat Bands?

Chapter 1:
What are Topological Materials?

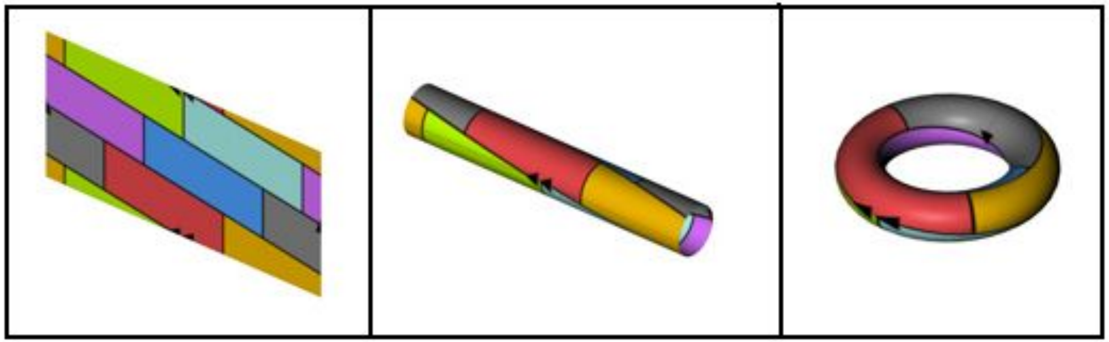
What are Topological Materials?

- Topological materials are materials which exhibit topological phases.
- These materials have promising applications in quantum computing, photonics, and electronic devices.
- What makes these phases topological? First, what even is topology?

Recall the old joke: a topologist cannot tell the difference between a coffee mug and a donut.

What are Topological Materials?

- Topology is the study of topological spaces up to *continuous deformations*, like stretching or bending.
- Topological materials thus have some physical property that is preserved under some notion of "stretching and bending."

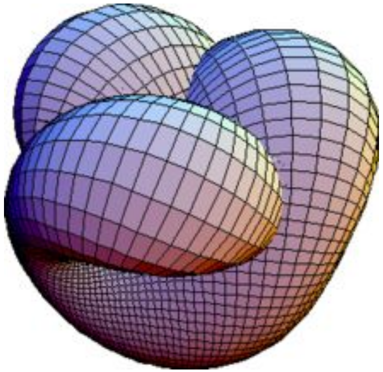


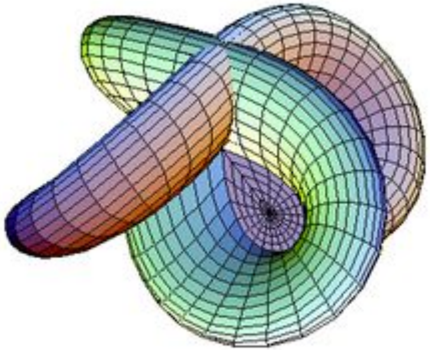
These are all toruses (the edges are glued).

Chapter 2:
Perspectives from Math

Perspectives from Math

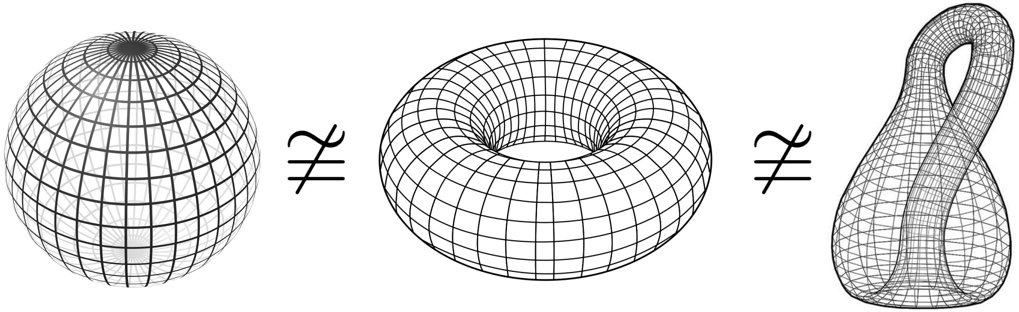
- In physics, we mainly deal with spaces called *manifolds*.
- Definition. A manifold is a topological space that is locally “the same as” a euclidean (“flat”) topological space.





Perspectives from Math

- Manifolds exhibit the dichotomy between *global and local properties*.
- *Locally*, all n-dimensional manifolds are the same.
- But clearly, manifolds can be different *globally* - a sphere is not a torus is not a klein bottle.



Perspectives from Math

- To help differentiate these spaces, math introduces *topological invariants* - global properties of topological spaces.
- To list a few, we have the fundamental group π_1, the homology/cohomology groups, and the ***Euler characteristic***.
- Physical systems also have *topological invariants* which depend on the topology of the system.

Chapter 3:
Perspectives from Physics

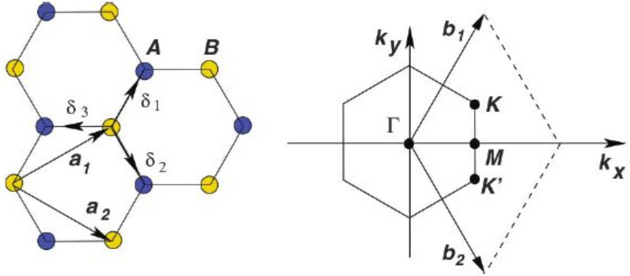
Perspectives from Physics

- Typically in physics, the state of a system is determined by *local* parameters (eg. T, P, n).
- However, in *topological phases*, the state of the system is determined by *topological invariants* of the system - these are *global* properties.
- Among these are the *Chern Number* and the Z_2 *invariant,* which relate to the electronic behavior of the material.

Perspectives from Physics

The Chern number is the integral of the *Berry curvature* over *filled bands* in the *Brillouin zone*.

- The Berry curvature is a tensor that contains information on how quantum states evolve in adiabatic transport.
- The Brillouin zone is the parameter space that indexes the eigenstates of a periodic Hamiltonian.



Perspectives from Physics

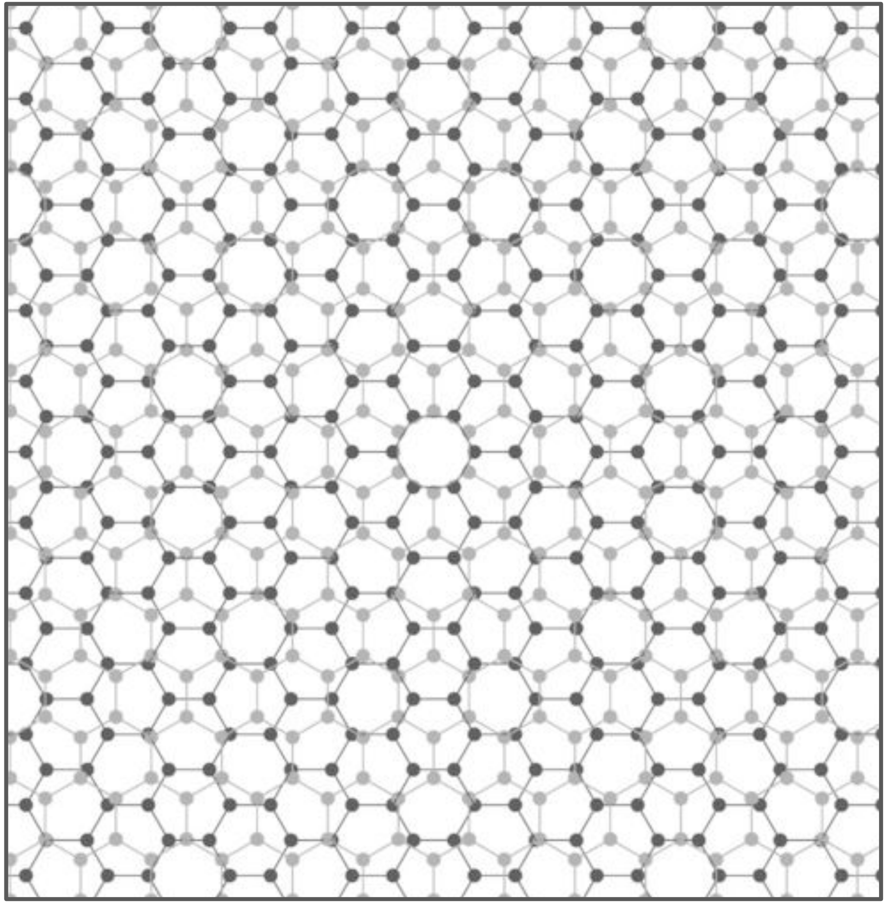
The Chern number is the integral of the *Berry curvature* over *filled bands* in the *Brillouin zone*.

- The Brillouin zone is a manifold.
- For a 2D lattice, it is periodic in two directions, making it a torus!

Perspectives from Physics

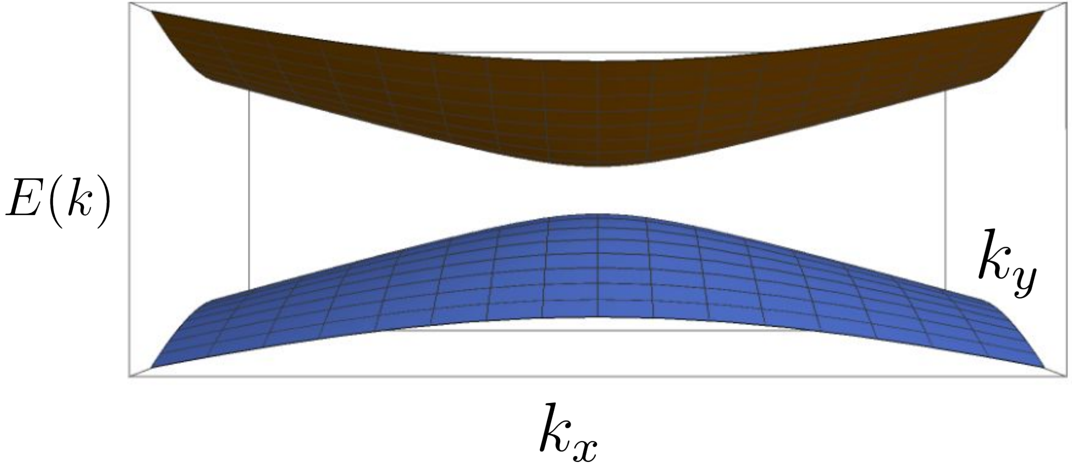
- This is important, as the integral of the curvature over a manifold is 2π times the *Euler characteristic* of the space (*Gauss–Bonnet theorem*).
- **So, the topology of the Brillouin zone will have physical implications on the behavior of the system.**
- Singularities of the Berry curvature can change the topology of the system and thus the Chern number.

Chapter 4:
What are Flat Bands?



What are Flat Bands?

- *Bands* in the Brillouin zone are collections of eigenstates with some energy relation $E(k)$.
- They tell us a lot of information about the material, like conductivity, optical absorption, and degeneracy points.



What are Flat Bands?

- *Flat bands* have a **constant** dispersion relation $E(k) = c$.
- This is interesting because without an energy dependence, the quantum geometry becomes more evident.
- Exciting topological phenomena are present in flat bands, like stable edge modes and superfluidity.
- Flat bands are experimentally achievable using *twisted graphene bilayers*.

References

Bernevig, B. Andrei. Topological Insulators and Topological Superconductors, Princeton: Princeton University Press, 2013. doi:10.1515/9781400846733

A. H. Castro Neto, F. Guinea, N. M. R. Peres, K. S. Novoselov, and A. K. Geim. The electronic properties of graphene. doi:10.1103/RevModPhys.81.109

Peotta, S., Törmä, P. Superfluidity in topologically nontrivial flat bands. Nat Commun 6, 8944 (2015). https://doi.org/10.1038/ncomms9944

Hatcher, A. Algebraic topology, Cambridge University Press, 2002, Cambridge.