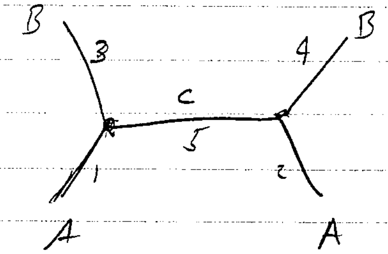
Notes : Henry Bradsher

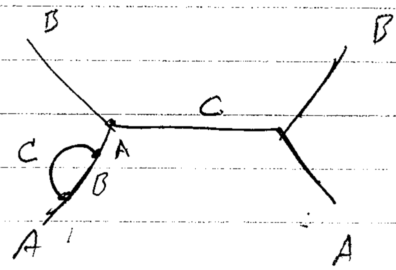
So far we've look at lowest order diagrams

Recall
2 vertices



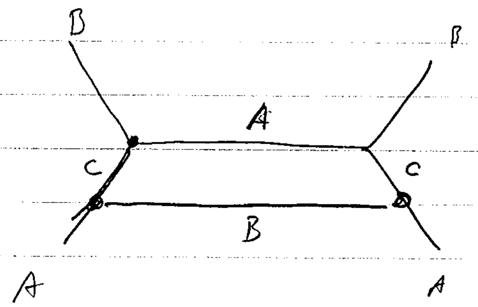
$M \propto g^2$

There are a number of diagrams with four vertices such as



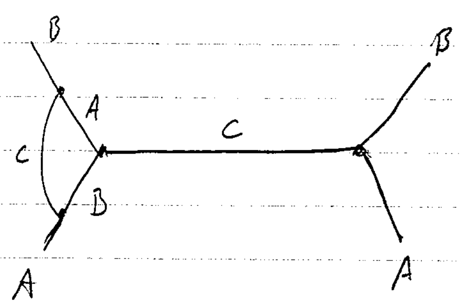
Starts : line 1
ends : line 1

or



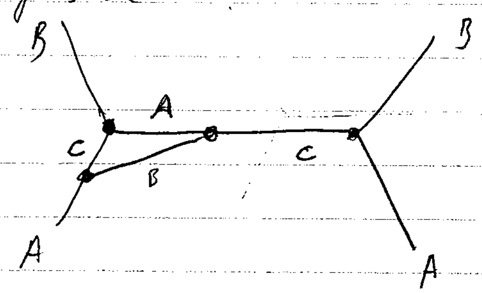
Starts : line 2
ends : line 2

or



Starts : line 1
ends : line 3

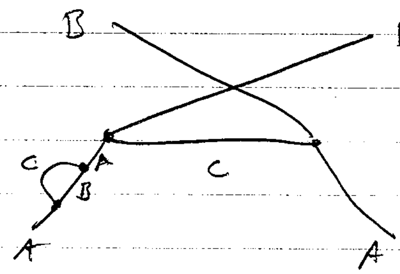
also possible



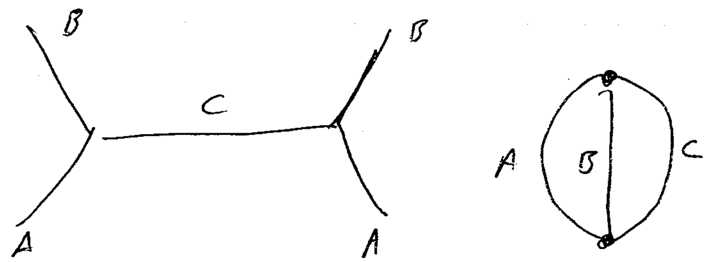
Starts : line 1
ends : line 5

Also 5 w/ added line attaches to line 2
but we already counted line 2 → line 2
so there is $5+4+3+2+1 = 15$
4th order diagrams.

Also another 15 for twisted version
such as

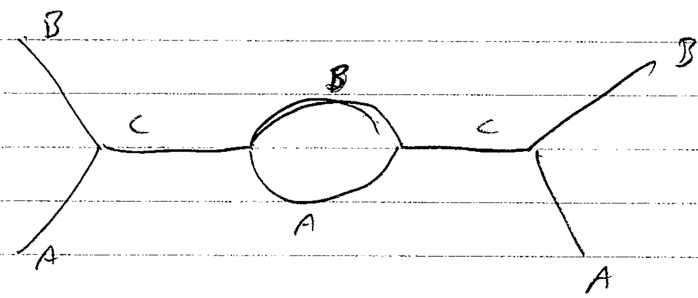


incidentally : disconnected diagrams such as

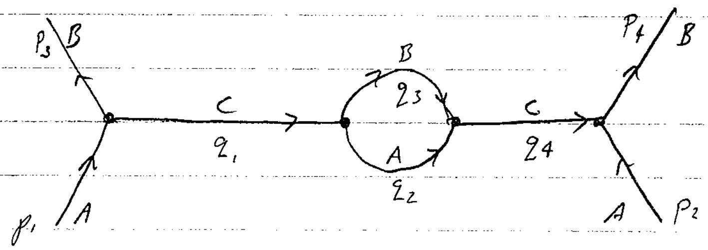


Don't Count

Takeing a closer look at



Using the 6 steps we 1st label



2nd: we write a factor $-ig$ for each vetex

$(-ig)^4$

3rd: we write a factor $\frac{i}{q_j^2 - m_j^2c^2}$ for each internal line

$$(-ig)^4\left(\frac{i}{q_1^2 - m_C^2c^2}\right)\left(\frac{i}{q_2^2 - m_A^2c^2}\right)\left(\frac{i}{q_3^2 - m_B^2c^2}\right)\left(\frac{i}{q_4^2 - m_C^2c^2}\right)$$

4th: For each vertex write a delta fxn of the form

$(2\pi)^4\delta^4(k_1 + k_2 + k_3)$

In this case

$$(2\pi)^4\delta^4(p_1 - q_1 - p_3)(2\pi)^4\delta^4(q_1 - q_2 - q_3)$$
$$(2\pi)^4(q_2 + q_3 - q_4)(2\pi)^4\delta(q_4 + p_2 - p_4)$$

5th: for each internal line write a factor

$\frac{1}{(2\pi)^4}d^4q_j$ and integrate

$$\left(\frac{1}{2\pi}\right)^4 g^4 \left[(p_1-p_3)^2 - m_c^2c^2\right]^{-1}\left[(p_4-p_2)^2 - m_c^2c^2\right]^{-1} \times$$

$$\int \frac{1}{[(p_1-p_3-q_3)^2 - m_A^2c^2][q_3^2 - m_B^2c^2]} \cancel{\delta(p_1+p_2-p_3-p_4)}\,(2\pi)^4\, d^4q_3$$

$$iM = \left(\frac{g}{2\pi}\right)^4 \left[(p_1-p_3)^2 - m_c^2c^2\right]^{-1}\left[(p_4-p_2)^2 - m_c^2c^2\right]^{-1}$$

$$\int \left[(p_1-p_3-q)^2 - m_A^2c^2\right]^{-1}\left[q^2 - m_B^2c^2\right]^{-1} dq$$

$$d^4q = q^3\, dq\, d\Omega'$$

$$1/q^4 \; q^3\, dq\, \cancel{d\Omega'}$$

$d\Omega' = 2\pi$: Angular part of Integral

$$dq/q = \ln q \Big]_0^\infty = \infty \longrightarrow$$ Held up QED for 2 decades

Introduce $-m^2c^2/(q^2 - m^2c^2)$

$$\lim_{m\to\infty} \to 1$$

$m_{physical} = m + \delta m$ $\quad g_{phys} = g + \delta g$

δm and δg go to infinity

We call these the renormalized masses + coupling constants

We can ignore the δm's + δg's $\to$ because we never measure them

This means that the effective masses + coupling constants depend on particle energies

- leads to a lamb shift in QED
- leads to asymptotic freedom in QCD

If all infinities can be accomodated in this way the theory is renormalizable

Non renormalizable theories yield answers that are cutoff dependant and meaningless