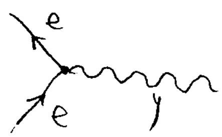
6.3 The Feynman Rules for A Toy Theory

From the previous section, we know how to calculate decay rates and scattering cross section, both of which mention the amplitude M for the process in question.

So how to determine M?

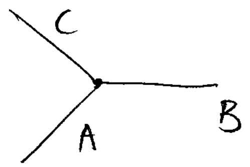
Smart Feynman introduced "Feynmen rules" to evaluate the relavant diagrames.

For example, electrons and photons interact via primitive vertex:

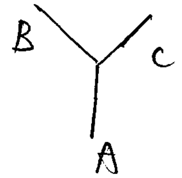


Well, it roughly show the process how e and γ interact, but, unfortunately, the process involves diverting complications, because electron carries spin ½ and the photon carries pin 1). However, fortunately, these complications have nothing to do with the Feynman calculus as such. The stuff involving particles with spin will be discussed in Chapter 7. As simple as possible, we'll talk about the "toy" theory, and it serves to illustrate the method to calculate M, but not pretend to represent the real world. It's always what we need to do as a physics student: we put forward a model study it, and apply it to something complicated, then we conclude from all the results and back to something easy again and again.

OK Now. Imagine a world made of three kinds of particles - A, B and C, masses m_A, m_B, and m_C. Spin 0, each is its own particle anti.
here's one primitive vertex.

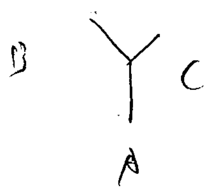


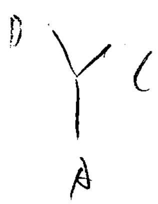
Assume, $m_A > m_B + m_C$,
Then, we have no choice but write



Because of the energy conservation.

The Lowest diagram describing this disintegration.
We treat it as toy bricks,
Many higher order interactions



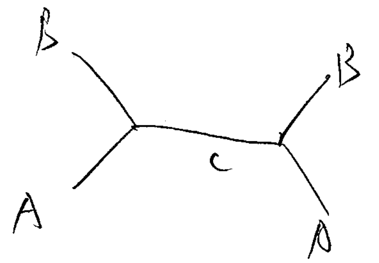


$\Rightarrow$ 2-nd order diagram

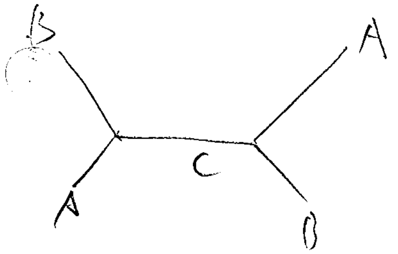
$\Rightarrow$ 3-rd order

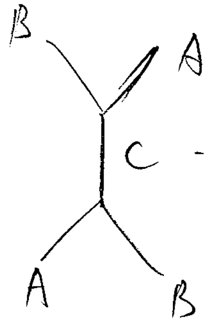
Consider

$$A + A \rightarrow B + B$$ Scattering process.



$A + B \to A + B$

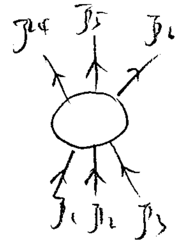




Relate it to Bhabha Scattering

The ritual method to find the amplitude M associated with a given diagram is as follows.

1. Notation.



arbitrarily assigned for the internal lines.

2. Coupling constant. for each vertex, factor $-ig$

g called coupling constant, specifies the strength of the interaction between A, B and C. In toy theory g has the dimension of momentum.

3. Propagator. For each internal line, factor

$$\frac{i}{q_j^2 - m_j^2 c^2}$$

q_j fourth vector. m_j is the mass of the particle the line describes.

4. Conservation of energy and momentum

For each vertex, $(2\pi)^4 \delta^4(k_1 + k_2 + k_3)$ k's are 3 4-momenta coming to the vertex.

If k is coming into the vertex, k has the "+" sign, or it has the "-" sign.

5. Integration over Internal Momenta.

For each internal line, $\frac{1}{(2\pi)^4} d^4 q_j$, and integrate over all internal momenta.

6. Cancel the Delta function.

The results will include a delta function

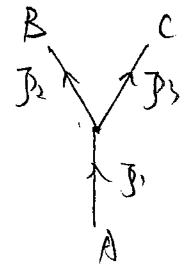
$$(2\pi)^4 \delta^4(p_1 + p_2 + \cdots - p_n)$$

enforcing overall conservation of energy and momentum. Erase this factor, and what remains is $-i\mathcal{M}$.

Then Let's see how to evaluate some elementary Feynman diagrams in the "ABC" theory

6.4 Lifetime of the A

$A \to B + C$



No internal lines

One vertex

Step 1. Done

Step 2. $-ig$

Step 3. ~~[illegible]~~ Discard

Step 4. $(2\pi)^4 \delta^4(p_1 - p_2 - p_3)$

Step 5. Discard

$$-i\mathcal{M} = -ig\,(2\pi)^4 \delta^4(p_1 - p_2 - p_3)$$

Step 6. $\mathcal{M} = g$ (amplitude to the lowest order).

(6.32)
$$\Gamma = \frac{g^2 |\vec{p}|}{8\pi \hbar m_A^2 c}$$

$|\vec{p}|$ magnitude of either outgoing momentum.

$$|\vec{p}| = \frac{c}{2m_A}\sqrt{m_A^4 + m_B^4 + m_C^4 - 2m_A^2 m_B^2 - 2m_A^2 m_C^2 - 2m_B^2 m_C^2}$$

$$\tau = \frac{1}{\Gamma}$$