Section 6.2:

(Sangeetha.)

Golden Rule for Scattering:

$$1+2 \longrightarrow 3+4+\cdots+n$$

Cross Section:

$$d\sigma = |\mathcal{M}|^2 \frac{\hbar^2 S}{4\sqrt{(p_1^\mu \cdot p_{2\mu})^2 - (m_1 m_2 c^2)^2}} \left[\frac{c\, d^3\vec{p}_3}{(2\pi)^3 2E_3} \frac{c\, d^3\vec{p}_4}{(2\pi)^3 2E_4} \cdots \frac{c\, d^3\vec{p}_n}{(2\pi)^3 2E_n} \right.$$

$$\times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4 \cdots - p_n)$$

$p_i = (E_i/c, \vec{p}_i)$ = four-momentum $\qquad S = \frac{1}{j!}$, for each group of j identical partic

$E_i = p_i^2 c^2 + m_i^2 c^4$

$\delta \longrightarrow$ energy & momentum conservation

$\frac{d\sigma}{d\Omega}$ = differential cross-section for the particle 3 into solid a

$\Rightarrow$ Integrate over all momenta (other than p_3) & over the magnit of p_3.

UNITS:

(i) Decay Rate $= \Gamma = \frac{1}{\tau}$, τ - lifetime $\Rightarrow$ units of time

$\Rightarrow [\Gamma]$ = inverse time

(ii) $\left[\frac{d\sigma}{d\Omega}\right] = \frac{\text{barns}}{\text{steradian}}$

(iii) $[\mathcal{M}] = (mc)^{4-n}$, n = number of external lines

example: $\quad A \rightarrow B+C \Rightarrow n=3 \Rightarrow [\mathcal{M}] = mc$ = dimensions of mome

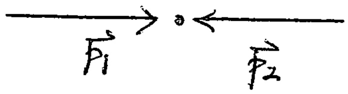
$A \longrightarrow B+C+D$

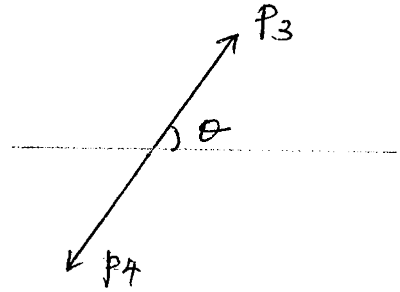
$A+B \longrightarrow C+D$ $\Big\} \Rightarrow n=4 \Rightarrow [\mathcal{M}] = 1$ = dimensionless.

2 - Body Scattering:

$$1+2 \longrightarrow 3+4$$

CM-frame:





$$d\sigma = |M|^2 \frac{\hbar^2 S}{4\sqrt{(p_1^{\mu}\cdot p_{2\mu})^2-(m_1 m_2 c^2)^2}} \left[\frac{c\, d^3\vec{p}_3}{(2\pi)^3\, 2E_3}\;\frac{c\, d^3\vec{p}_4}{(2\pi)^3\, 2E_4}\right](2\pi)^4\delta^4(p_1+p_2-p_3-$$

In CM-frame:

$$\vec{p}_2 = -\vec{p}_1 \Rightarrow p_1^{\mu}\cdot p_{2\mu} = p_1^0 p_2^0 - \vec{p}_1\cdot\vec{p}_2$$

$$= \frac{E_1}{c}\frac{E_2}{c} - \vec{p}_1(-\vec{p}_1)$$

$$= \frac{E_1E_2}{c^2} + |p_1|^2$$

$$\Rightarrow (p_1^{\mu}\cdot p_{2\mu})^2 - (m_1 m_2 c^2)^2 = \left[\frac{E_1E_2}{c^2} + |p_1|^2\right]^2 - (m_1 c\, m_2 c)^2$$

$$= \frac{E_1^2}{c^2}\frac{E_2^2}{c^2} + p_1^4 - \underbrace{m_1^2c^2m_2^2c^2}_{-\left[\left(\frac{E_2^2}{c^2}-p_2^2\right)m_1^2c^2\right]} + 2\frac{E_1E_2}{c^2}p_1^2$$

$$p_2^2 = p_1^2$$

$$= \frac{E_1^2}{c^2}\frac{E_2^2}{c^2} - \frac{E_2^2}{c^2}m_1c^2 + p_1^4 + p_1^2m_1^2c^2 + 2\frac{E_1E_2}{c^2}p$$

$$= \frac{E_2^2}{c^2}\Big[\underbrace{\frac{E_1^2}{c^2} - m_1c^2}_{p_1^2}\Big] + p_1^2\Big[\underbrace{p_1^2 + m_1^2c^2}_{E_1^2/c^2}\Big] + 2\frac{E_1E_2}{c^2}p$$

$$= (\underbrace{E_2^2 + E_1^2 + 2E_1E_2}_{(E_1+E_2)^2})\frac{p_1^2}{c^2}$$

$$\Rightarrow \sqrt{(p_1\cdot p_2) - (m_1m_2c^2)^2} = (E_1+E_2)\frac{|p_1|}{c} \longrightarrow ①$$

$$\delta^4(p_1+p_2-p_3-p_4) = \delta^1\delta^3$$
$$= \delta\left(\frac{E_1+E_2-E_3-E_4}{c}\right)\delta^3(\underbrace{\vec{p}_1+\vec{p}_2}_{\vec{p}_1+\vec{p}_2=0}-\vec{p}_3-\vec{p}_4)$$

E_3 & E_4 $\Rightarrow E^2 = p^2c^2 + mc^4$
$$= c^2(p^2+mc^2)$$

& $\delta^4(p_1+p_2-p_3-p_4) = \delta\left[(E_1+E_2)/c - \sqrt{m_3^2c^2+p_3^2} - \sqrt{m_4^2c^2+p_4^2}\right]\delta^3(\vec{p}$

Constants:
$$\frac{\hbar^2}{4} \times \frac{c^2}{4(2\pi)^6} \times (2\pi)^4 = \left(\frac{\hbar}{8\pi}\right)^2 \cdot c^2$$

$\int$ ing $d\vec{p}_4 \Rightarrow \vec{p}_4 \longrightarrow -\vec{p}_3$ & writing $d\vec{p}_3 = p_3^2 dp_3 d\Omega$

$$\frac{d\sigma}{d\Omega} = S\left(\frac{\hbar}{8\pi}\right)^2 \cdot c^2 \frac{1}{(E_1+E_2)\frac{|p_1|}{c}} \int_0^\infty \frac{|M|^2}{c^2\sqrt{p_3^2+m_3c^2}\ \sqrt{p_3^2+m_4c^2}}$$

← function of p_3, cannot go out of integral

$$\cdot\,\delta\left[(E_1+E_2)/c - \sqrt{m_3^2c^2+p_3^2} - \sqrt{m_4^2c^2+p_4^2}\right] p_3^2\, dp_3$$

Compare with (6.25)

(i) $p_2 \longrightarrow p_3$

(ii) $\delta[m_1c \ldots\ldots] \longrightarrow \delta[(E_1+E_2)/c \ \cdots]$

(iii) let $E \equiv c\left[\sqrt{m_3^2c^2+\rho^2} + \sqrt{m_4^2c^2+\rho^2}\right]$

$\int d\rho = \frac{\rho_0}{m_1 c}$ where ρ_0 is a particular value for $E = m_1c^2$

$\Rightarrow$ for this case $\longrightarrow E = E_1 + E_2$

all that integral yields is $\longrightarrow \frac{|p_2|}{m_1 c}|M|^2$

$$\Rightarrow \frac{d\sigma}{d\Omega} = \left(\frac{\hbar^2}{8\pi}\right)^2 S \cdot \frac{1}{(E_1+E_2)\frac{p_1}{c}} \frac{|p_3||M|^2}{(E_1+E_2)/c} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|M|^2|\vec{p}_f|}{(E_1+E_2)^2|\vec{p}_i|}$$

Note: on M:

$M = M(p_1, p_2, p_3, p_4)$

$$\left.\begin{array}{l}\vec{p}_2 = -\vec{p}_1 \\ \vec{p}_4 = -\vec{p}_3\end{array}\right\} M = M(\vec{p}_1, \vec{p}_3) = M(p_1, p_3, \theta)$$