Archana

Sec. 6.2

Presentation - Phys 5213.

Golden Rule for Decays

For decay processes, the most important quantity that we need to determine is the 'DECAY RATE'.

Decay Rate - The probability per unit time that a given particle will decay. - 'Γ'

$$dN = -\Gamma N dt.$$

Equation obeyed by decay rate, with dN number of particles decaying in time dt.

The mean lifetime of a particle

$$\tau = \frac{1}{\Gamma}.$$

(1) Amplitude - contains all dynamical information about a process. It is determined using the Feynman Rules.

(2) Phase Space - Contains all kinematical information and relates mass, energy, momenta, etc.,

These two quantities are necessary to

to calculate the transition rate for any process.

Fermi Golden Rule:

$$\text{Transition Rate} = \frac{2\pi}{\hbar} |M|^2 \times (\text{phase space}).$$

Decay: $1 \rightarrow 2 + 3 + \dots n.$

The differential decay rate is given by:

$$d\Gamma = |M|^2 \frac{S}{2\hbar m_1} \left[\left(\frac{c\, d^3 p_2}{(2\pi)^3 2E_2} \right) \left(\frac{c\, d^3 p_3}{(2\pi)^3 2E_3} \right) \cdots \left(\frac{c\, d^3 p_n}{(2\pi)^3 2E_n} \right) \right]$$

$$\times (2\pi)^4 \delta^4 (p_1 - p_2 - p_3 \cdots p_n).$$

$$p_i = \left(\frac{E_i}{c}, \vec{p}_i \right) \quad ; \quad E^2 = p^2 c^2 + m^2 c^4.$$

δ function makes sure that energy/momentum are conserved.

For all the cases that we will consider:

$$p_1 = (m_1 c, 0) \quad (\text{Particle at rest})$$

S: statistical factor.

$= \frac{1}{j!}$ for each group of j particles in the final state

Example:

$$\pi^0 \to \gamma + \gamma \qquad S = 1/2.$$

Now, the total decay rate is: (for a 2 particle final state).

$$\Gamma = \frac{S}{\hbar m_1}\left(\frac{c}{4\pi}\right)^2 \frac{1}{2}\int \frac{|\mathcal{M}|^2}{E_2 E_3}\,\delta^4(p_1 - p_2 - p_3)\,d^3p_2\,d^3p_3.$$

DECAY INTO MASSLESS SECONDARIES:

Eg: $\pi^0 \to \gamma + \gamma.$

Amplitude $\mathcal{M} \equiv \mathcal{M}(p_2, p_3).$

$p_1 = (mc, 0) \qquad m_2 = 0 = m_3$

We can first decompose the delta function as:

$$\delta^4(p_1 - p_2 - p_3) = \delta\left(mc - \frac{E_2}{c} - \frac{E_3}{c}\right)\delta^3(-\mathbf{p}_2 - \mathbf{p}_3)$$

$$= \delta(mc - |\mathbf{p}_2| - |\mathbf{p}_3|)\,\delta^3(-\mathbf{p}_2 - \mathbf{p}_3).$$

Since $m_2 = 0 = m_3$;

$E_2 = |\mathbf{p}_2| c; \quad E_3 = |\mathbf{p}_3| c.$

Now the total decay rate becomes:

$$\Gamma = \frac{S}{\hbar m}\left(\frac{1}{4\pi}\right)^2 \frac{1}{2}\int \frac{|M|^2}{|p_2||p_3|}\,\delta(mc - |p_2| - |p_3|)\,\delta^3(p_2 - p_3)\, d^3p_2\, d^3p_3$$

$$= \frac{S}{\hbar m}\left(\frac{1}{4\pi}\right)^2 \frac{1}{2}\int \frac{|M|^2}{|p_2|^2}\,\delta(mc - 2|p_2|)\, d^3|p_2|$$

$$= \frac{S}{\hbar m}\left(\frac{1}{4\pi}\right)^2 \frac{1}{2}\int \frac{|M|^2}{|p_2|^2}\,\delta(mc - 2|p_2|)\,|p_2|^2 d|p_2| \sin\theta\, d\theta\, d\phi$$

$$= \frac{S}{8\pi\hbar m}\int |M|^2\,\delta(mc - 2|p_2|)\, d|p_2|$$

We use the two properties of δ-function to simplify the above:

(i) $\delta(-x) = \delta(x)$

(ii) $\delta(ax) = \frac{1}{|a|}\delta(x)$

$$\therefore \Gamma = \frac{S}{8\pi\hbar m}\int |M|^2 \frac{1}{2}\,\delta\left(|p_2| - \frac{mc}{2}\right) d|p_2|.$$

$$\Gamma = \frac{S}{16\pi\hbar m}|M|^2.$$

Decay rate for: decay into massless secondaries.

Two body decay:

$p_1 = (m_1 c, 0)$ Outgoing masses: m_2, m_3.

Now:

$$\Gamma = \frac{S}{\hbar m_1}\left(\frac{c}{4\pi}\right)^2 \frac{1}{2} \int \frac{|M|^2}{E_2 E_3} \delta^4(p_1 - p_2 - p_3)\, d^3p_2\, d^3p_3.$$

$$E_2 = c\sqrt{m_2^2 c^2 + p_2^2} \quad \& \quad E_3 = c\sqrt{m_3^2 c^2 + p_3^2}.$$

Now,

$$\delta^4(p_1 - p_2 - p_3) = \delta\left(m_1 c - \frac{E_2}{c} - \frac{E_3}{c}\right)\delta^3(-p_2 - p_3).$$

$$\Gamma = \frac{S}{\hbar m_1}\left(\frac{c}{4\pi}\right)^2 \frac{1}{2} \int \frac{|M|^2}{E_2 E_3} \delta\left(m_1 c - \frac{E_2}{c} - \frac{E_3}{c}\right)\delta^3(-p_2 - p_3)\, d^3p_2\, d^3p_3.$$

$$= \frac{S}{\hbar m_1}\left(\frac{c}{4\pi}\right)^2 \frac{1}{2} \int \frac{|M|^2 \delta\left(m_1 c - \sqrt{m_2^2 c^2 + p^2} - \sqrt{m_3^2 c^2 + p_2^2}\right) d^3p}{\sqrt{m_2^2 c^2 + p^2}\sqrt{m_3^2 c^2 + p_2^2}}$$

Doing the angular integration, we get a factor of 4π.

Now we do a variable change:

$$E = c\left(\sqrt{m_2^2 c^2 + p^2} + \sqrt{m_3^2 c^2 + p^2}\right)$$

$|p_2| = p.$

$$dE = c\left[\frac{1 \cdot \cancel{2} p\, dp}{\cancel{2}\sqrt{m_2^2c^2+p^2}} + \frac{\cancel{2} p\, dp}{\cancel{2}\sqrt{m_3^2c^2+p^2}}\right)$$

$$= \frac{c\left(\sqrt{m_2^2c^2+p^2} + \sqrt{m_3^2c^2+p^2}\right) p\, dp}{\sqrt{m_2^2c^2+p^2}\,\sqrt{m_3^2c^2+p^2}}$$

$$dE = \frac{E p\, dp}{\sqrt{m_2^2c^2+p^2}\,\sqrt{m_3^2c^2+p^2}}.$$

$$\Gamma = \frac{S}{8\pi\hbar m_1}\int_{(m_2+m_3)c^2}^{\infty} |M|^2\, \delta\left(m_1c - \frac{E}{c}\right)\frac{p}{E}\, dE.$$

Note that the lower bound cannot be zero.

$$\Gamma = \frac{S}{8\pi\hbar m}\int |M|^2 c\, \delta\left(E - m_1c^2\right)\frac{p}{E}\, dE$$

(Using properties of δ-function)

Hence

$$\Gamma = \frac{S|M|^2 p_0}{8\pi\hbar m_1^2 c} \quad \text{for two-body decay.}$$

$$p_0 = \frac{c}{2m}\sqrt{m_1^4+m_2^4+m_3^4-2m_1^2m_2^2-2m_1^2m_3^2-2m_2^2m_3^2}.$$