Ch 6.1 Lifetimes & ~~Crosse~~ Cross Sections

Scattering & Decays are particularly well described by by relativistic theory, in Feynman's formulation.

For Decays:

What do we want to know about the decay?
We ~~want to know the~~ would like to know the lifetime of the particles.

So intested in rest frame life time

This is effected by ~~th~~ time dialation from relativity. So particles travelling at different speeds will have different relative lifetimes.
Particle decay can be repesented as independant of time. In any given dt, the particle is equally likely to decay.
So we need the decay rate:
The probability of decay per unit time.

Decayed in a given dt.

Then we have $\boxed{dN = -\Gamma N\, dt}$ decay,
where Γ is the decay rate
& $N(t)$ is the number of particles.

because few particles as they decay

$$\frac{dN}{dt} = -\Gamma N$$

$$\Rightarrow \boxed{N(t) = N(0)e^{-\Gamma t}}$$

So decay is exponential

lifetime (τ) ~~defined~~ as:

using the decay rate we can find the lifetime as (τ)

$$\boxed{\tau = \frac{1}{\Gamma}}$$

Most particles have multiple decay routes, so the total decay rate would be

sum since different possible routes to decay

$$\Gamma_{total} = \sum_{i=1}^{n} \Gamma_i \qquad ?$$

and the corresponding τ (lifetime)

$$\tau = \frac{1}{\Gamma_{tot}} = \frac{1}{\sum_{i=1}^{n} \Gamma_i}$$

Now we want the branching ratios:

These are defined as

percentage decayed in ith route

$$\frac{\Gamma_i}{\Gamma_{total}}$$

The branching ratio is the ratio of particles that decay via the 'i'th' route.

Scattering:

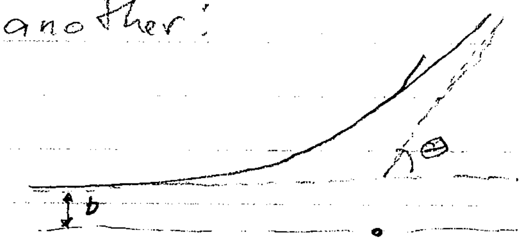
We need to determine a cross section which defined the effective scattering area. These processes that contribute can be electromagnetic, weak, and strong.

Thus $\sigma_{tot} = \sum_{1}^{n} \sigma_i$

we would think that $\uparrow v \Rightarrow$ smaller effect

Each σ_i has a dependance on velocity. Such that at certain energies it hits a resonance. These resonances are used to find short lived particles.

When two particles approach one another:



where $\theta \equiv$ scattering angle
and $b \equiv$ impact parameter

$$\theta = \theta(b)$$

If we vary b by db, then

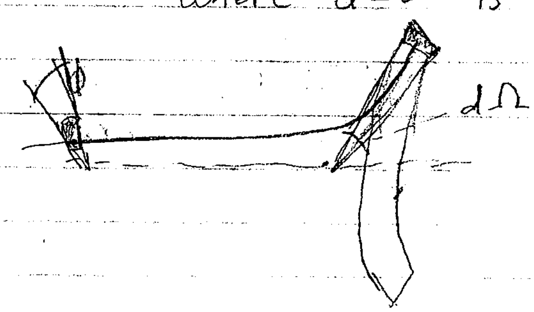
$$b_1 = b + db$$
$$\theta_1 = \theta + d\theta$$

insert the next example →

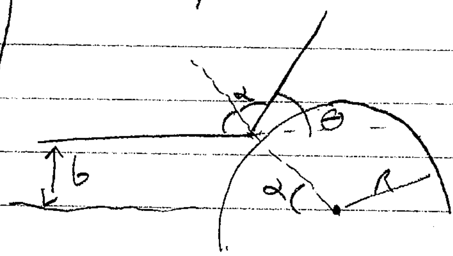
so in general, if the particle goes through some $d\sigma$, then

$$d\theta = \frac{d\theta}{d\Omega} d\Omega$$

where $d\Omega$ is the solid scattering angle



Ex. Where we scatter off a hard sphere.



$$b = R\sin\alpha$$

$$2\alpha + \theta = \pi$$

$$b = R\sin\alpha = R\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = R\cos\left(\frac{\theta}{2}\right)$$

$$\frac{b}{R} = \cos\left(\frac{\theta}{2}\right)$$

$$2\cos^{-1}\left(\frac{b}{R}\right) = \theta$$

Note: then

$$b \to b + db$$

$$d\theta = 2\left(\arccos\left(\frac{b+db}{R}\right) - \arccos\left(\frac{b}{R}\right)\right)$$

$\frac{d\sigma}{d\Omega}$ differential cross section

$$d\sigma = \frac{d\sigma}{d\Omega}d\Omega$$

$$= |b\,db\,d\phi|$$

$$d\Omega = |\sin\theta\, d\theta\, d\phi|$$

get from previous figure

$$\frac{d\sigma}{d\Omega} = \left|\frac{b}{\sin\theta}\frac{db}{d\theta}\right|$$

just deal with positives

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

Ex Hard Sphere extension:

$$\frac{db}{d\theta} = -\frac{R}{2}\sin\left(\frac{\theta}{2}\right)$$

so
$$\frac{d\sigma}{d\Omega} = \frac{Rb\sin(\theta/2)}{2\sin\theta} = \frac{R^2}{2}\,\frac{\cos(\frac{\theta}{2})\sin(\frac{\theta}{2})}{\sin\theta} = \boxed{\frac{R^2}{4}}$$

$$\sigma = \int d\sigma = \int \frac{d\sigma}{d\Omega}\,d\Omega$$

follows from $d\sigma = \frac{d\sigma}{d\Omega}\,d\Omega$

Ex Hard Sphere (cont)

$$\frac{d\sigma}{d\Omega} = \frac{R^2}{4}$$

$$\sigma = \int \frac{R^2}{4}\,d\Omega = \cancel{\pi R^2}$$

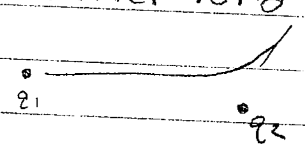
$$= \int_0^{2\pi}\int_0^{\pi} \frac{R^2}{4}\sin\theta\,d\theta\,d\phi = \int_0^{\pi} \frac{\pi R^2}{2}\sin\theta\,d\theta$$

$$= -\cos(\theta)\,\frac{\pi R^2}{2}\Big|_0^{\pi} = \cancel{2}\,\frac{\pi R^2}{\cancel{2}} = \boxed{\pi R^2}$$

$\sigma = \pi R^2 =$ cross sectional area of the sphere

So, as expected, any particles outside that area are uneffected.

Ex Rutherford Scattering



$$b = \frac{q_1 q_2}{2E}\cot\left(\frac{\theta}{2}\right)$$ $E =$ initial T of q_1

$$\frac{d\sigma}{d\Omega} = \left(\frac{q_1 q_2}{4E\sin^2(\theta/2)}\right)^2$$

$$\sigma = 2\pi\left(\frac{q_1 q_2}{4E}\right)^2 \int_0^{\pi} \frac{1}{\sin^4 \theta/2} \sin\theta \, d\theta = \infty$$

The Coulomb potential has infinet range thus the cross section is infinet.

For a beam of luminosity ($\mathcal{L}$) then:

$$dN = \mathcal{L}\, d\sigma = \mathcal{L} \frac{d\sigma}{d\Omega} d\Omega$$

$$\rightarrow \frac{d\sigma}{d\Omega} = \frac{dN}{d\Omega}\frac{1}{\mathcal{L}}$$

~~# of particles scattered per unit time into the solid angle $d\Omega$~~

event rate = cross section × luminosity
This assumes a stationary target.