

ERROR SPEED OF LIGHT

$$C = \frac{8\pi AD^2 (f_1 + f_2)}{D+B (s_1 - s_2)}$$

remember $Q = f(a, b, c)$

$$\sigma_{MQ}^2 = \left(\frac{\partial Q}{\partial a}\right)^2 \sigma_{ma}^2 + \left(\frac{\partial Q}{\partial b}\right)^2 \sigma_{mb}^2$$

or

$$\left(\frac{\sigma_{MQ}}{Q}\right)^2 = \left(\frac{1}{Q} \frac{\partial Q}{\partial a}\right)^2 \sigma_{ma}^2 + \dots$$

Now in $Q = a^n b^m c \dots$

$$\text{then } \left(\frac{\sigma_Q}{Q}\right)^2 = n^2 \left(\frac{\sigma_a}{a}\right)^2 + m^2 \left(\frac{\sigma_b}{b}\right)^2$$

BUT c above is not like this

Why? $D+B \neq f_1 + f_2 \neq s_1 - s_2$

We can always define new "simplifying" VARIABLES

CHAIN RULE for ERROR

$$\text{eg } F = f_1 + f_2, \quad S = s_1 - s_2, \quad E = D + B$$

and look at

$$\frac{\sigma_F}{F} = \frac{\sigma_E}{E}$$

①

$$F = S_1 + S_2$$

$$\sigma_F^2 = \left(\frac{\partial F}{\partial S_1}\right)^2 \sigma_{S_1}^2 + \left(\frac{\partial F}{\partial S_2}\right)^2 \sigma_{S_2}^2$$

$$\left(\frac{\sigma_F}{F}\right)^2 = 2 \left(\frac{\sigma_S}{S}\right)^2$$

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$$S = S_1 - S_2$$

$$\sigma_S^2 = \left(\frac{\partial S}{\partial S_1}\right)^2 \sigma_{S_1}^2 + \left(\frac{\partial S}{\partial S_2}\right)^2 \sigma_{S_2}^2$$

$$\left(\frac{\sigma_S}{S}\right)^2 = 2 \left(\frac{\sigma_S}{S}\right)^2 \left. \begin{array}{l} \text{Big Contribution} \\ \text{SMALL} \end{array} \right\}$$

$$E = D + B$$

NOTE HOW E is not independent of D or B
Could proceed with

$$\left(\frac{\sigma_E}{E}\right)^2 = \left(\frac{\sigma_D}{D}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2$$

So that, contribution of $\frac{D^2}{D+B}$ on $\left(\frac{\sigma_C}{C}\right)^2$

$$\frac{4\left(\frac{\sigma_D}{D}\right)^2}{D^2} + \frac{\left(\frac{\sigma_D}{D}\right)^2}{D+B} + \left(\frac{\sigma_B}{B}\right)^2$$

$$= 5\left(\frac{\sigma_D}{D}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2$$

But better use $\sqrt{G} = \frac{D^2}{D+B}$

$$\frac{\partial G}{\partial D} = 2D \left(\frac{1}{D+B} \right) - \frac{D^2}{(D+B)^2} = \frac{2D^2 + 2DB - D^2}{(D+B)^2} = \frac{D^2 + 2DB}{(D+B)^2}$$

$$\frac{\partial G}{\partial B} = - \frac{D^2}{(D+B)^2}$$

$$\left(\frac{\sigma_G}{G} \right)^2 = \frac{(D^2 + 2DB)^2}{(D+B)^4} \sigma_D^2 + \frac{D^4}{(D+B)^4} \sigma_B^2$$
