Solutions to Homework Set #7
Phys2414 – Fall 2005

Note: The numbers in the $\boxed{\text{boxes}}$ correspond to those that are generated by WebAssign. The numbers on your individual assignment will vary. Any calculated quantities that involve these variable numbers will be $\boxed{\text{boxed}}$ as well.

1. GRR1 6.P.002. A sled is dragged along a horizontal path at a constant speed of 1.5 m/s by a rope that is inclined at an angle of 30.0° with respect to the horizontal (the figure below). The total weight of the sled is 470 N. The tension in the rope is $\boxed{230}$ N. How much work is done by the rope on the sled in a time interval of $\boxed{5.0}$ s?

The formula for calculating the work done on an object is

$$W = |\vec{F}| d \cos\theta$$

where $|\vec{F}|$ is the magnitude of the force applied, d is the distance traveled, and $\cos\theta$ is the cosine of the angle between the force vector and the displacement vector. In this case the displacement is horizontal and equal to the speed times the time interval given, i.e.

$$d = vt$$

The work done is then given by

$$\begin{aligned} W &= |\vec{F}|\,(vt)\cos\theta \\ W &= (\boxed{230}\ \text{N}) \cdot (1.5\ \text{m/s}) \cdot (\boxed{5.0}\ \text{s}) \cdot \cos(30.0^\circ) \\ W &= 1.49\ \text{kJ} \end{aligned}$$

2. GRR1 6.P.005. A barge of mass 5.0×10^4 kg is pulled along the Erie Canal by two mules, walking along towpaths parallel to the canal on either side of it. The ropes harnessed to the mules make angles of 45° to the canal. Each mule is pulling on its rope with a force of 1.0 kN. How much work is done on the barge by both of these mules together as they pull the barge $\boxed{130}$ m along the canal?

The total work done is just the sum of the work done by each individual force.

$$W_{total} = \sum_i W_i$$

The work done by a force is given by

$$W_F = |\vec{F}| d \cos\theta$$

So the work done by a single mule is given by

$$W_{mule} = (1.0 \text{ kN}) \cdot (\boxed{130} \text{ m}) \cdot \cos(45°) = \boxed{92} \text{ kN}$$

So the total work done is just the work done by each mule

$$W_{total} = W_{mule} + W_{mule} = 2W_{mule}$$
$$W_{total} = \boxed{184} \text{ kN}$$

3. GRR1 6.P.012. A plane weighing $\boxed{220}$ kN (25 tons) lands on an aircraft carrier. The plane is moving horizontally at $\boxed{64}$ m/s ($\boxed{143}$ mi/h) when its tailhook grabs hold of the arresting cables. The cables bring the plane to a stop in a distance of $\boxed{83}$ m.

(a) How much work is done on the plane by the arresting cables?
Work total work done is equal to the change in kinetic energy. Since the arresting cables are the only force that does work on the plane, then the work done by the arresting cables is the total work. So the work done by the arresting cables is equal to the change in kinetic energy.

$$W = \Delta K$$
$$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$m = W/g$$
$$\Delta K = \frac{1}{2}m(0)^2 - \frac{1}{2}(\boxed{220} \text{ kN}/9.81 \text{ m/s}^2) \cdot (\boxed{64} \text{ m/s})^2$$
$$W = \boxed{-45.9} \text{ MJ}$$

(b) What is the force (assumed constant) exerted on the plane by the cables?
The work done by a force is given by:

$$W = |\vec{F}| d \cos\theta$$

Solving for the force $|\vec{F}|$ we have the following formula. We plug in our previously calculated value for work, and because the cables pull on the plane in the opposite direction of the plane's motion we use 180° for the angle.

$$|\vec{F}| = \frac{W}{d\cos\theta} = \frac{\boxed{-45.9}\text{ MJ}}{(\boxed{83}\text{ m}\cos(180°)}$$
$$|\vec{F}| = \boxed{553}\text{ kN}$$

4. GRR1 6.P.021. Justin moves a desk $\boxed{4.8}$ m across a level floor by pushing on it with a constant horizontal force of $\boxed{310}$ N. (It slides for a negligibly small distance before coming to a stop when the force is removed.) Then, changing his mind, he moves it back to its starting point, again by pushing with a constant force of $\boxed{310}$ N.

(a) What is the change in the desk's gravitational potential energy during the round-trip?
The change in gravitational potential energy is given by the following formula. Since the initial and final positions are the same $(h_f - h_i) = 0$. So the change in gravitational potential energy is.

$$\Delta U_g = mgh_f - mgh_i = mg(h_f - h_i) = mg(0) = 0$$

(b) How much work has Justin done on the desk?
The total work is the sum of all the individual works

$$W_{total} = \sum_i W_i$$

The work done on a single trip is then:

$$W_{singletrip} = |\vec{F}|d\cos\theta = (\boxed{310}\text{ N})\cdot(\boxed{4.8}\text{ m})\cos(0°)$$
$$W_{singletrip} = \boxed{1.49}\text{ kJ}$$

So the total work done is just the sum of the two single trips.

$$W_{total} = 2W_{Singletrip} = \boxed{2.98}\text{ kJ}$$

(c) If the work done by Justin is not equal to the change in gravitational potential energy of the desk, then where has the energy gone?
Friction is a non-conservative force, so the mechanical energy is not conserved. The energy is dissipated in the form of heat

5. GRR1 6.P.028. When a 0.20 kg mass is suspended from a vertically hanging spring, it stretches the spring from its original length of $\boxed{3.0}$ cm to a total length of $\boxed{7.0}$ cm. The spring with the same mass attached is then placed on a horizontal frictionless surface. The mass is pulled so that the spring stretches to a total length of 10.0 cm; then the mass is released and it oscillates back and forth (the figure below). What is the maximum speed of the mass as it oscillates?

This problem has two parts, one where the spring is hanging from the ceiling and another where the spring is on a frictionless horizontal surface. We use the information of how the spring behaves as you hang is from a ceiling in to get the spring constant k. We use Newtons second law on the mass to do this.

$$\sum \vec{F} = m\vec{a}$$

The only forces acting on the spring are the force of the spring, and weight. Solving the equation for k gives us:

$$F_{spring} - mg = 0$$
$$k = \frac{mg}{x} = \frac{(.2 \text{ kg}) \cdot (9.81 \text{ m/s}^2)}{(\boxed{7.0} - \boxed{3.0}) \text{ cm}}$$
$$k = \boxed{49} \text{ N/m}$$

In the second part we use the conservation of energy to find the maximum velocity of the block. The conservation of energy states:

$$\Delta K + \Delta U_s = W_{NC}$$

Where ΔK is the change in kinetic energy, ΔU is the change in potential energy, and W_{NC} is the work done by non-conservative forces. We then plug in $K = 1/2mv^2$ and $U_s = 1/2kx^2$, and since there is no friction there is no work done by non-conservative forces or W_{NC} is zero.

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + \frac{1}{2}kx_f^2 - \frac{1}{2}mx_i^2 = 0$$

We know the initial velocity is zero, and solving for the final velocity we get.

$$v_f^2 = \frac{k}{m}(x_i^2 - x_f^2)$$
$$v_f = \sqrt{\frac{k}{m}(x_i^2 - x_f^2)}$$

So if we want the final velocity to be a maximum, then obviously x_f must be zero. We can then plug in our value for the spring constant k we calculated, the mass, and the initial stretch of the spring (which would be the stretched length 10 cm minus the unstretched length $\boxed{3}$ cm.

$$v_f = \sqrt{\frac{k}{m}}(x_i)$$
$$v_f = \sqrt{\frac{\boxed{49}\ \text{N/m}}{0.2\ \text{kg}}}((10 - \boxed{3.0})\ \text{cm})$$
$$v_f = \boxed{1.1}\ \text{m/s}$$

6. GRR1 6.P.029. A cart starts from position 4 in the figure below with a velocity of $\boxed{13}$ m/s to the left. Find the speed with which the cart reaches positions 1, 2, and 3. Neglect friction.

We want to start with the conservation of energy.

$$\Delta K + \Delta U_g = W_{NC}$$

We then plug in $K = 1/2mv^2$ and $U = mgh$, and use the fact that there is no friction so $W_{NC} = 0$. We can then solve for v_f.

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i = 0$$
$$v_f = \sqrt{v_i^2 + 2g(h_i - h_f)}$$

speed at position 1

For these parts we just plug in the values of initial velocity v_i, initial height h_i and final height h_f.

$$v_f = \sqrt{(\boxed{13}\ \text{m/s})^2 + 2(9.81\ \text{m/s}^2) \cdot (20 - 0)\ \text{m}}$$
$$v_f = \boxed{23.7}\ \text{m/s}$$

speed at position 2

$$v_f = \sqrt{(\boxed{13}\ \text{m/s})^2 + 2(9.81\ \text{m/s}^2) \cdot (20 - 15)\ \text{m}}$$
$$v_f = \boxed{16.3}\ \text{m/s}$$

speed at position 3

$$v_f = \sqrt{(\boxed{13}\ \text{m/s})^2 + 2(9.81\ \text{m/s}^2)\cdot(20-10)\ \text{m}}$$
$$v_f = \boxed{19.1}\ \text{m/s}$$

7. GRR1 6.P.034. An object slides down an inclined plane of angle $\boxed{30.0}^\circ$ and of incline length $\boxed{2.0}$ m. If the initial speed of the object is 6.0 m/s directed down the incline, what is the speed at the bottom? Neglect friction.

We can do this problem like problem # 6. We start with the conservation of energy, plug in expressions for kinetic energy K and potential energy U. There is no friction so $W_{NC} = 0$. We then solve for v_f.

$$\Delta K + \Delta U_g = W_{NC}$$
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i = 0$$
$$v_f = \sqrt{v_i^2 + 2g(h_i - h_f)}$$

Next we have to realize that the change in height $(h_i - h_f)$ is related to the distance the block travels down the incline d.

$$h_i - h_f = d\sin\theta = (\boxed{2.0}\ \text{m})\cdot\sin\left(\boxed{30}^\circ\right) = \boxed{1}\ \text{m}$$

Finally we just plug in values.

$$v_f = \sqrt{(6.0\ \text{m/s})^2 + 2(9.81\ \text{m/s}^2)\cdot(\boxed{1}\ \text{m})}$$
$$v_f = \boxed{7.46}\ \text{m/s}$$

8. GRR1 6.P.039. A spring with k = $\boxed{10.0}$ N/m is at the base of a frictionless 30.0 inclined plane. A 0.50 kg object is pressed against the spring, compressing it $\boxed{0.2}$ m from its equilibrium position. The object is then released. If the object is not attached to the spring, how far up the incline does it travel before coming to rest and then sliding back down? (See the figure below.)

We start with the conservation of energy equation, where for this problem we include both a gravitational potential energy term U_g and a spring potential energy term U_s.

$$\Delta K + \Delta U_g + \Delta U_s = W_{NC}$$

Without friction we know that $W_{NC} = 0$. We also know that since our initial and final velocities are zero, then the total change in kinetic energy is zero.

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - 0 = 0$$

Plugging in expressions for ΔU_g and ΔU_s we get the following formula.

$$mgh_f - mgh_i + \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 = 0$$

We then have to relate the change in height to the distance traveled up the incline d:

$$h_f - h_i = d \sin\theta$$

We then just solve for the distance up the incline d.

$$d = \frac{kx_i^2}{2mg\sin\theta} = \frac{(\boxed{10}\text{ N/m})(\boxed{0.2}\text{ m})^2}{2 \cdot (0.50\text{ kg}) \cdot (9.81\text{ m/s}^2) \cdot \sin(30°)}$$
$$d = \boxed{0.082}\text{ m}$$

9. GRR1 6.P.044. A spring gun (k = $\boxed{28}$ N/m) is used to shoot a 56 g ball horizontally. Initially the spring is compressed by $\boxed{19}$ cm. The ball loses contact with the spring and leaves the gun when the spring is still compressed by 12 cm. What is the speed of the ball when it hits the ground, $\boxed{1.4}$ m below the spring gun?

We start with the conservation of energy. Then plug in the expressions for ΔK, ΔU_g, and ΔU_s, and since there is no friction or air resistance we can set $W_{NC} = 0$.

$$\Delta K + \Delta U_g + \Delta U_s = W_{NC}$$
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i + \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 = 0$$

Since the ball starts from rest we set the initial velocity v_i to zero, and solve for the final velocity. We can then plug in the given values for the change in height $h_i - h_f$, the spring constant k, the mass m, and the initial and final values of the spring compressions x_i and x_f.

$$v_f = \sqrt{2g(h_i - h_f) + \frac{k}{m}(x_i^2 - x_f^2)}$$
$$v_f = \sqrt{2 \cdot (9.81\ [m/s^2])(\boxed{1.4} - 0)\text{ m} + \frac{\boxed{28}\text{ N/m}}{.056\text{ kg}}((\boxed{.19}\text{ m})^2 - (\boxed{.12}\text{ m})^2)}$$
$$v_f = \boxed{6.19}\text{ m/s}$$

10. GRR1 6.P.057. The power output of a cyclist moving at a constant speed of $\boxed{5.0}$ m/s on a level road is $\boxed{110}$ W.

(a) What is the force exerted on the cyclist and the bicycle by the air?
Power is given by the formula $P = Fv$, where P is power, F is the force, and v is the velocity. So using that formula and solving for F gives:

$$\begin{aligned} P &= Fv \\ F = P/v &= \boxed{110}\text{ W}/\boxed{5.0}\text{ m/s} \\ F &= \boxed{22}\text{ N} \end{aligned}$$

(b) By bending low over the handlebars, the cyclist reduces the air resistance to $\boxed{16}$ N. If she maintains a power output of $\boxed{110}$ W, what will her speed be?
Solving the power formula for velocity gives:

$$\begin{aligned} P &= Fv \\ v = P/F &= \boxed{110}\text{ W}/\boxed{26}\text{ N} \\ F &= \boxed{6.9}\text{ m/s} \end{aligned}$$

11. GRR1 6.P.078. [299744] A 0.50 kg block, starting at rest, slides down a 30.0 incline with kinetic friction coefficient $\boxed{0.28}$ (the figure below). After sliding $\boxed{83}$ cm down the incline, it slides across a frictionless horizontal surface and encounters a spring (k = $\boxed{33}$ N/m).

(a) What is the maximum compression of the spring?
We should use the conservation of energy to solve this problem, so we start out with the conservation of energy equation.

$$\Delta K + \Delta U_g + \Delta U_s = W_{NC}$$

First we look at the change in kinetic energy ΔK. Since the initial and final velocities are both zero, then the change in kinetic energy is also zero.

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - 0 = 0$$

We then plug in expressions for ΔU_g and ΔU_s, and since there is friction we replace W_{NC} with the work done by friction W_f.

$$mgh_f - mgh_i + \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 = W_f$$

But we need to figure out what the work done by friction is. The formula for work done by a force is:

$$W_f = |\vec{F}_f| d \cos(180°)$$

But now we need to find the force of friction. We start by use the expression the expression for force of kinetic friction (kinetic since the block is moving). $F_f = \mu_k F_N$. Drawing a free body diagram, and setting the axis up along the plane we get for the sum of forces perpendicular to the plane.

$$F_N - mg\cos(30°) = 0$$

So the force of friction is then given by the following formula, and the work done by friction is then similarly given by the next formula.

$$F_f = \mu_k mg\cos(30°)$$
$$W_f = (\boxed{0.28}) \cdot (0.50 \text{ kg}) \cdot (9.81 \text{ m/s}^2)\cos(30°) \cdot (\boxed{83} \text{ cm}) \cdot \cos(180°) = \boxed{-0.99} \text{ J}$$

Plugging this into our equation for conservation of energy, and setting $x_i = 0$ and $(h_i - h_f) = d\cos(30°)$, we get:

$$\frac{1}{2}kx_f^2 = mgd\sin(30°) + W_f = (0.50 \text{ kg}) \cdot (9.81 \text{ m/s}^2) \cdot (\boxed{83} \text{ cm})\sin(30°) - \boxed{-0.99} \text{ J}$$
$$\frac{1}{2}kx_f^2 = \boxed{1.04} \text{ J}$$
$$x_f = \sqrt{2 \cdot (\boxed{1.04} \text{ J})/(\boxed{33} \text{ N/m})}$$
$$x_f = \boxed{25} \text{ cm}$$

(b) After the compression of part (a), the spring rebounds and shoots the block back up the incline. How far along the incline does the block travel before coming to rest?
Again we want to use the conservation of energy to solve this problem. And again for the same reason as in part a we want to set $\Delta K = 0$.

$$\Delta K + \Delta U_g + \Delta U_s = W_{NC}$$
$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - 0 = 0$$

So plugging in the usual expressions for ΔU_g and ΔU_s and setting $W_{NC} = W_f$ we get:

$$mgh_f - mgh_i + \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 = W_f$$

Again we replace the change in height with the distance up the plane times the sine of the angle of the plane, and we use the formula for work done by a force.

$$h_f - h_i = d \sin\theta$$
$$W_f = |\vec{F}_f| d \cos(180°)$$

And finally we solve for d the distance up the plane the block slides.

$$mgd\sin\theta + |\vec{F}_f|d = \frac{1}{2}kx_i^2$$
$$d = \frac{\frac{1}{2}kx_i^2}{mg(\sin\theta + \mu_k\cos\theta)} = \frac{\boxed{1.04}\ \text{J}}{(0.50\ \text{kg})\cdot(9.81\ \text{m/s}^2)\cdot(\sin(30°) + \boxed{0.28}\cos(30°))}$$
$$d = \boxed{28.5}\ \text{cm}$$

12. GRR1 6.TB.042. [219697] A roller coaster car (mass = 988 kg including passengers) is about to roll down a track (the figure below). The diameter of the circular loop is 20.0 m and the car starts out from rest 40.0 m above the lowest point of the track. Ignore friction and air resistance and assume g = 9.81 m/s2.

(a) At what speed does the car reach the top of the loop?
We start with the conservation of energy equation, and plug in the usual expressions for ΔK and ΔU_g, and set $W_{NC} = 0$ since there is no friction.

$$\Delta K + \Delta U_g = W_{NC}$$
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i = 0$$

We then solve for the final velocity.

$$v_f = \sqrt{v_i^2 + 2g(h_i - h_f)}$$
$$v_f = \sqrt{(0)^2 + 2(9.81\ \text{m/s})(40 - 20)\ \text{m}}$$
$$v_f = 19.8\ \text{m/s}$$

(b) What is the force exerted on the car by the track at the top of the loop?
The question ask about a force, so we start with a free body diagram, the free body diagram only has two forces: mg pointing straight down, and F_N also pointing down since the track is above the cart at the top of the loop. Newton's Second law then states:

$$\sum \vec{F} = m\vec{a}$$

But we know that the acceleration is the centripetal acceleration, so $a = v^2/r$. Solving for F_N gives:

$$F_N + mg = \frac{mv^2}{r}$$
$$F_N = \frac{mv^2}{r} - mg = \frac{(988 \text{ kg}) \cdot (19.8 \text{ m/s})^2}{10 \text{ m}} - (988 \text{ kg}) \cdot (9.81 \text{ m/s})$$
$$F_N = 29.1 \text{ kN}$$

(c) From what minimum height above the bottom of the loop can the car be released so that it does not lose contact with the track at the top of the loop?
To do this problem we first want to find the minimum velocity the car must have to stay on the track on the top of the loop, and then use the conservation of energy to find from what height the cart must be released so that it reaches that velocity.

To find from what the minimum velocity needed for the cart to stay on the track we must use the newtons second law again, but let the normal force just go to zero.

$$F_N + mg = \frac{mv^2}{r}$$

So letting $F_N = 0$ and solving for v^2 gives us:

$$v^2 = gr$$

Now we want to use the conservation of energy to find h_i, if $v_f^2 = gr$.

$$\Delta K + \Delta U_g = W_{NC}$$
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh_f - mgh_i = 0$$
$$mgh_i = mgh_f + \frac{1}{2}mv_f^2 = mgh + \frac{1}{2}mgr$$
$$h_i = h_f + \frac{1}{2}r = 20 \text{ m} + \frac{1}{2}10 \text{ m}$$
$$h_i = 25 \text{ m}$$