

Read 11.7-11.12

Evaluations ~20%

No group this week

Action Center Thursday

Bonus H.W Due Friday

Office hours

10:30-11:30 today

Final Exam: Wednesday

10:30-12:30 Here

Simple Harmonic Motion

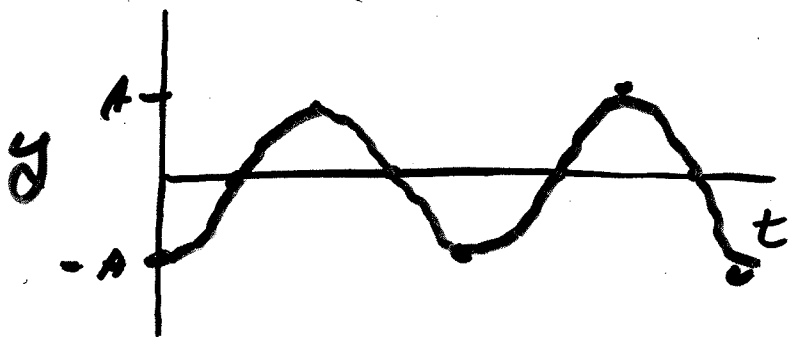
Vibrations and waves occur in many areas
sound, microwaves, light, ...

simplest to understand is called
simple Harmonic Motion

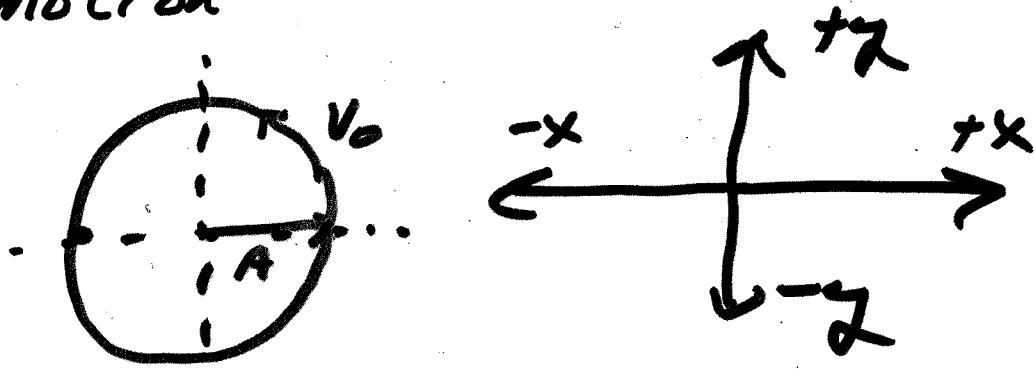
This is the motion of a spring with a
mass displaced from equilibrium

Spring $F = kx$

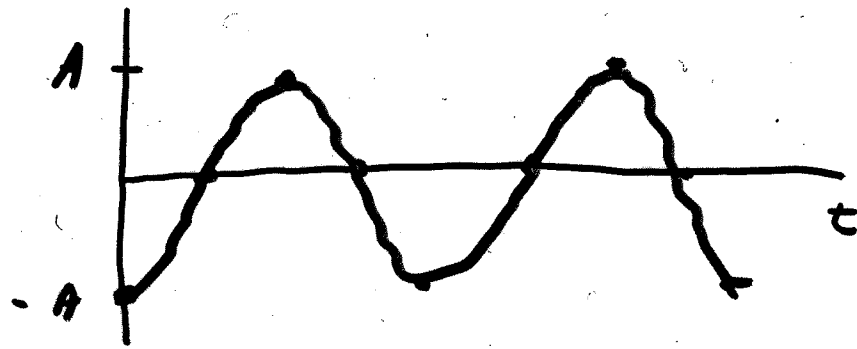
Let's plot position as a function
of time



We want to understand this motion
To do this, let's look at uniform circular
motion



Lets plot x position as function of
time



They look the same so let's apply
what we have learned previously
to understand simple harmonic motion

Circular motion

$$T = \frac{\text{Distance}}{\text{Velocity}} = \frac{2\pi A}{v_0}$$

period)

$$f = \frac{1}{T} \quad f = \frac{v_0}{2\pi A}$$

simple Harmonic motion

Let's use energy to find relationship between velocity and amplitude of oscillation

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

(K) (U)

at maximum displacement (Amplitude)

$$v = 0 \quad E = \frac{1}{2} k A^2$$

at maximum velocity, ~~amplitude~~ $x = 0$

$$E = \frac{1}{2} m v_0^2$$

$v_0 = \text{max velocity}$

$$\Rightarrow \text{Energy conserved} \Rightarrow \frac{1}{2} k A^2 = \frac{1}{2} m v_0^2$$

$$v_0^2 = \frac{k}{m} A^2$$

$$v_0 = \sqrt{\frac{k}{m}} A$$

put this into equation for the period

$$T = \frac{2\pi A}{v_0} = \frac{2\pi A}{\sqrt{\frac{k}{m}} A} = \boxed{2\pi \sqrt{\frac{m}{k}} = T}$$

$$f = \frac{1}{T} = \boxed{\frac{1}{2\pi} \sqrt{\frac{k}{m}} = f}$$

$x = A \cos \theta$ for uniform circular motion

rotating with angular velocity ω

$$\theta = \omega t$$

$$\omega = 2\pi f$$

$$\boxed{x = A \cos \omega t = A \cos 2\pi f t}$$

ex] Suppose I have a spring which oscillates according to the following equation

$$x = (0.25 \text{ m}) \cos\left(\frac{\pi t}{8.0}\right)$$

what is the amplitude, frequency and period of oscillation?

$$x = A \cos \underline{2\pi f t}$$

$$A = .25 \text{ m}$$

$$2\pi f t = \frac{\pi t}{8.0} \Rightarrow f = \frac{1}{2 \cdot 8.0} = \frac{1}{16} \text{ Hz}$$

$$\text{Hz} = \frac{1}{s}$$

$$T = \frac{1}{f} = \underline{16 \text{ s}}$$

Interactive Question

A mass vibrates back and forth from the free end of an ideal spring ($k = 20 \text{ N/m}$) with an amplitude of 0.30 m .

What is the kinetic energy of this vibrating mass when it is 0.30 m from its equilibrium position?

- A) Zero
- B) 1.80 J
- C) 0.90 J
- D) 0.45 J
- E) It is impossible to know without knowing the object's mass.

Interactive Question

Doubling only the amplitude of a vibrating mass and spring system produces what effect on the system's mechanical energy?

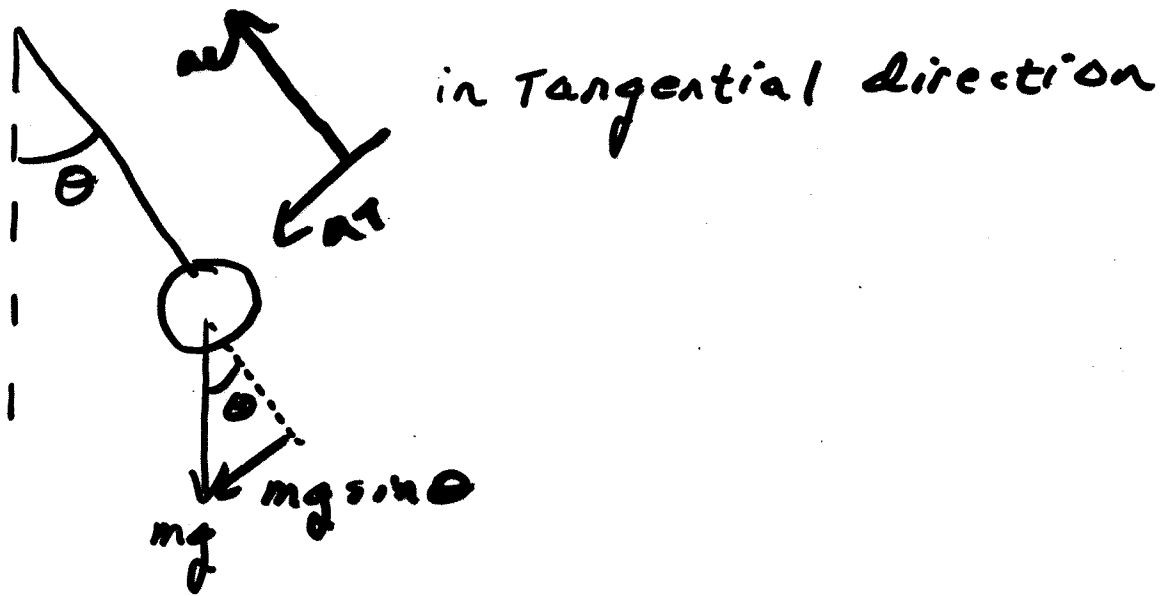
- A) It decreases the energy by a factor of four.
- B) It decreases the energy by a factor of two.
- C) It increases the energy by a factor of two.
- D) It increases the energy by a factor of four.
- E) It produces no change.

Interactive Question

A mass is attached to an ideal spring. When it is stretched a distance x , the system vibrates with a frequency f . In order to increase the frequency, one would have to

- A) reduce the spring constant.
- B) increase the length of the spring.
- C) decrease the mass on the end of the spring.
- D) reduce the distance that the spring is initially stretched.
- E) increase the distance that the spring is initially stretched.

Pendulum



$$F_T = mg \sin \theta$$

if θ small, $\sin \theta \approx \theta$, $x \approx l$



$$\sin \theta = \frac{x}{L}$$

$$F = \frac{mgx}{L}$$

$$F = kx$$

This looks like $F = kx$ if $k = \frac{mg}{L}$

for small angles, a pendulum behaves as a simple harmonic oscillator with a "spring constant"

$$\frac{mg}{L}$$

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{\frac{mg}{L}}} = 2\pi \sqrt{\frac{L}{g}}$$

period of a pendulum

$$T = 2\pi \sqrt{\frac{L}{g}}$$

only depends on length of pendulum
not the mass

Final Exam

Wednesday May 8 10:30-12:30

"Double" midterm

33 Questions

2 free

$\approx \frac{1}{2}$ concept $\approx \frac{1}{2}$ calculational

comprehensive

angular motion

torque

equilibrium

angular momentum

} more
emphasis

fluids, simple harmonic motion

2-3 questions