Read 9.1-9.2

H.W Due today

Next H.W Available

If you missed an exam
makeup Tuesday. talk/email
me if any questions.

Conservation of <u>angular momentum</u>

We know linear momentum ($\vec{p}$) is conserved

Let's look at angular momentum

Linear	Angular	
Force (F)	Torque (τ)	$x \leftrightarrow \theta$
		$v \leftrightarrow \omega$
		$a \leftrightarrow \alpha$
Kinetic Energy $\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	$m \leftrightarrow I$
Linear momentum $\vec{p} = m\vec{v}$	angular momentum $L = I\vec{\omega}$	
$\Sigma \vec{F} = m\vec{a} = \frac{\Delta \vec{p}}{\Delta t}$	$\Sigma \vec{\tau} = I\vec{\alpha} = \frac{\Delta \vec{L}}{\Delta t}$	
linear momentum conserved if no net external forces	angular momentum conserved if no net external torques	
$\Sigma \vec{p}_i = \Sigma \vec{p}_f$	$\Sigma \vec{L}_i = \Sigma \vec{L}_f$	
	$\Sigma I_i\omega_i = \Sigma I_f\omega_f$	
	$\Sigma m r_i^2 \omega_i = \Sigma m r_f^2 \omega_f$	

Interactive Question

An ice skater performs a pirouette by pulling her outstretched arms close to her body. What happens to her angular momentum about the axis of rotation?

A) It does not change.
B) It increases.
C) It decreases.
D) It changes, but it is impossible to tell which way.

Interactive Question

An ice skater performs a pirouette by pulling her outstretched arms close to her body. What happens to her moment of inertia about the axis of rotation?

A) It does not change.
B) It increases.
C) It decreases.
D) It changes, but it is impossible to tell which way.

Interactive Question

An ice skater performs a pirouette by pulling her outstretched arms close to her body. What happens to her rotational kinetic energy about the axis of rotation?

A) It does not change.
B) It increases.
C) It decreases.
D) It changes, but it is impossible to tell which way.

$K.E._i = \frac{1}{2} I_i \omega_i^2$

$K.E._f = \frac{1}{2} I_f \omega_f^2$

$I_i \omega_i = I_f \omega_f$ conservation of angular momentum

$$I_f = \frac{I_i \omega_i}{\omega_f}$$

$$K.E._f = \frac{1}{2} \frac{I_i \omega_i}{\omega_f} \omega_f^2$$

$K.E._f = \frac{1}{2} I_i \omega_i \omega_f$ ✓

$K.E._i = \frac{1}{2} I_i \omega_i^2$ ✓

$\omega_f > \omega_i$ so $K.E._f > K.E._i$

why is it increasing?

work $= \Delta K = F d \cos\theta$

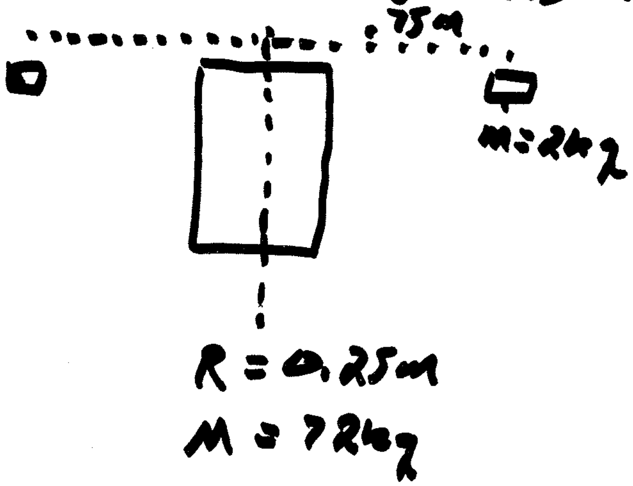
am I doing work?

yes

Force → centripetal

d in same direction as Force

ex) A student is sitting on a swivel seat and holding a 2kg mass in each hand. He is rotating at 1 rev/s when arms outstretched .75 m from axis of rotation. What happens when he pulls in arms. Person can be approximated as a cylinder of mass 72 kg and radius .25 m.



Conserve angular momentum

$L_i = L_f$

2 weights

$L_i = I_B \omega_i + 2 I_w \omega_i$

$L_f = I_B \omega_f + 0 \; (r=0, I=0)$

$I_B \omega_i + 2 I_w \omega_i = I_B \omega_f$ $\qquad I_B = \frac{1}{2} M R^2$

$(I_B + 2 I_w)\omega_i = I_B \omega_f$ $\qquad I_w = m r^2$

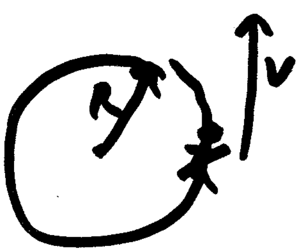
$$\omega_f = \left(\frac{\frac{1}{2} M R^2 + 2 m r^2}{\frac{1}{2} M R^2} \right) \omega_i$$

$$\omega_f = \left(\frac{\frac{1}{2}(72\,kg)(.25\,m)^2 + 2 \cdot (2\,kg)(.75\,m)^2}{\frac{1}{2}(72\,kg)(.25\,m)^2} \right)(1\,rev/s)$$

$$\boxed{\omega_f = 2\,rev/s}$$

ex) A girl of mass 20kg stands on the edge of a frictionless merry-go-round of radius 10m and $I = 500\ kg\,m^2$. She throws a rock of mass 1kg tangent to merry go round at a speed of 2 m/s.

What is angular speed of merry-go-round?



conserve angular momentum

$$L_i = L_f \qquad v = \omega r$$

$$I_i \omega_i = I_f \omega_f \qquad \omega = \frac{v}{r}$$

$$mR^2 \frac{v}{R}$$

$$mRv = (I_g + I_{merry})\,\omega_f$$

(I_g ↑ $m_g R^2$)

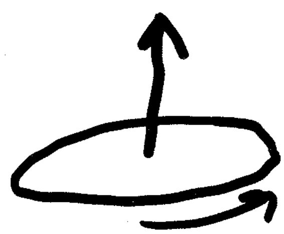
$$\omega_f = \frac{mRv}{(m_g R^2 + I_{merry})}$$

$$\omega_f = \frac{(1kg)(10m)(2m/s)}{(20kg)(10m)^2 + 500\,kg\text{-}m^2} = \underline{0.008\ rad/s}$$

Note Angular momentum is a vector and so has a direction.

Direction can be determined using right hand rule

Fingers in direction of rotation
thumb points in direction of vector



$$\Sigma \vec{\tau} = \frac{\Delta \vec{L}}{\Delta t}$$

Need a net torque to change angular momentum. So need a net torque to change direction of L.

→ stability of bicycle ??