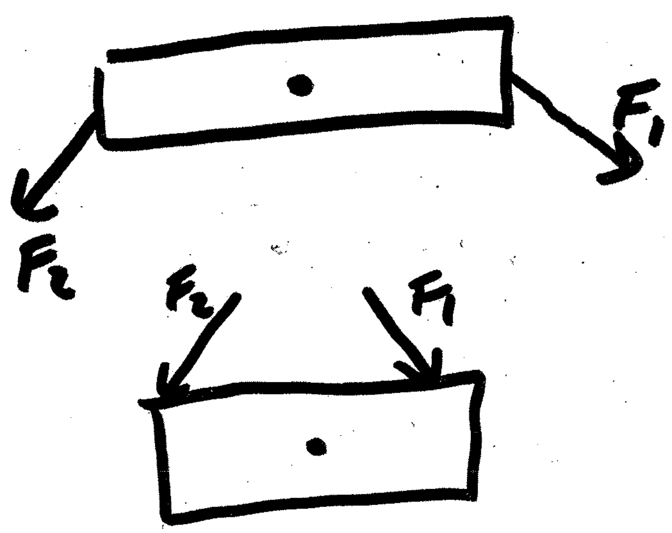
Read 8.8-8.9

H.W Due next Friday

Exam 3 Monday 7:30 a.m - 9:20 a.m
HERE. Chp 6-7

D2L updated with quiz 9

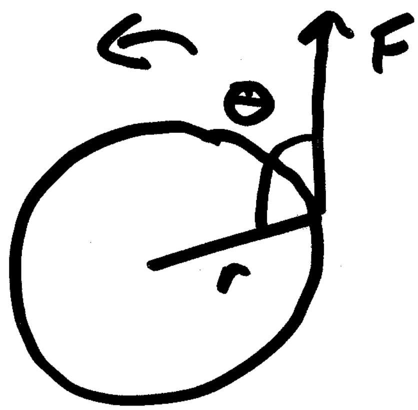


Last Lecture

introduced Torque (τ)

$\tau = r F \sin\theta$

<u>Torques</u> give rise to <u>rotations</u>

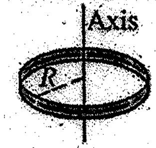
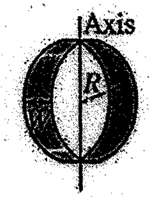
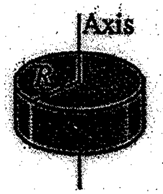
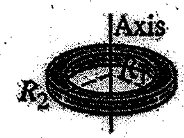
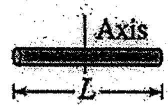
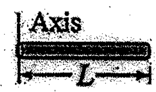


$\vec{\tau}_{net} = I\vec{\alpha}$ $\quad(\vec{F}_{net} = m\vec{a})$

$I = \Sigma m r^2$ (mass on a string) $I = mr^2$

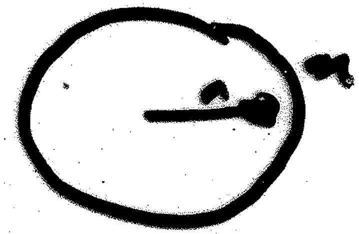
$I \equiv$ moment of Inertia

I depends on mass, shape and axis of rotation

	Object	Location of axis		Moment of inertia
(a)	**Thin hoop,** radius R	Through center		MR^2
(b)	**Thin hoop,** radius R width W	Through central diameter		$\frac{1}{2}MR^2 + \frac{1}{12}MW^2$
(c)	**Solid cylinder,** radius R	Through center		$\frac{1}{2}MR^2$
(d)	**Hollow cylinder,** inner radius R_1 outer radius R_2	Through center		$\frac{1}{2}M(R_1^2 + R_2^2)$
(e)	**Uniform sphere,** radius R	Through center		$\frac{2}{5}MR^2$
(f)	**Long uniform rod,** length L	Through center		$\frac{1}{12}ML^2$
(g)	**Long uniform rod,** length L	Through end		$\frac{1}{3}ML^2$
(h)	**Rectangular thin plate,** length L, width W	Through center		$\frac{1}{12}M(L^2 + W^2)$

Point mass spinning

$I = mr^2$



$F = ma$ $\tau = mr^2\alpha$

$F \to \tau$

$a \to \alpha$

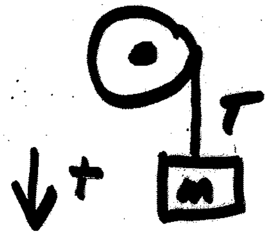
$m \to mr^2$ (I)

ex) A cylindrical <u>3.0 kg pulley</u> with radius $R = 0.4\,m$ is used to lower a 2.0 kg bucket. The bucket starts from rest and falls for 3.0 s

a) what is linear acceleration of bucket

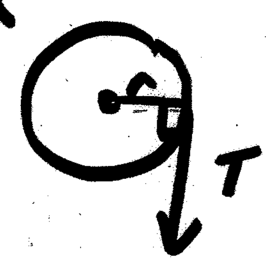
b) How far does it drop

c) what is angular acceleration of pulley



FBD

pulley



$\tau = rF\sin\theta$

$\theta = 90^\circ$

$\tau = rF = rT$

$rT = I\alpha$

$a_T = \alpha r$

$\alpha = \frac{a_T}{r}$

$\Sigma F = ma$

$\Sigma \tau = I\alpha$

① $-T + mg = ma$

$rT = I\alpha = \left(\frac{1}{2}Mr^2\right)\alpha = \frac{1}{2}Mr^2\frac{a}{r}$

② $T = \frac{1}{2}Ma$

algebra

(2) $T = \frac{1}{2} M a$

put into (1)

M = pulley mass
m = bucket mass

$$mg - \frac{1}{2} M a = m a$$

$$a = \left(\frac{mg}{m + \frac{M}{2}} \right) = \frac{(2\,kg)(9.8\,m/s^2)}{2\,kg + \frac{3\,kg}{2}} = \underline{5.6\,m/s^2}$$

$$M = 0 \quad a = \frac{mg}{m + 0} = \underline{g}$$

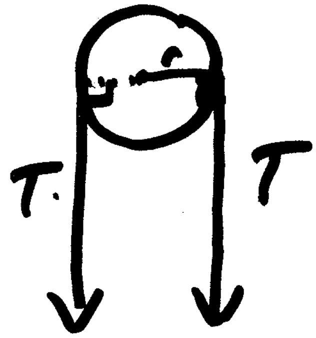
b) $y = y_0 + v_{0y}^{\,0} t + \frac{1}{2} a t^2$

$$y - y_0 = 0 + \frac{1}{2}(5.6\,m/s^2)(3\,s)^2 = \underline{25.2\,m}$$

c) $\alpha = \frac{a}{R} = \frac{5.6\,m/s^2}{0.4\,m} = 14.0\,rad/s^2$

massless pulley

$\tau = rT$



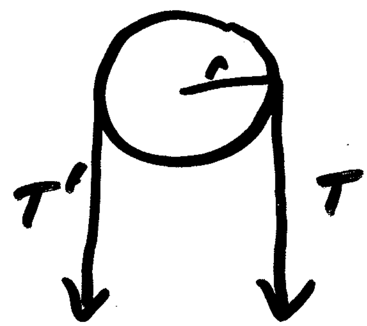
$\tau = -rT$

$rT + (-rT) = 0$

$\tau_{net} = I\alpha$

$I = 0$ $\quad$ $0 = 0$

massive pulley



$I = \frac{1}{2}mr^2$

need a net torque for a massive pulley

T' must be different than T so have a net torque so can have angular acceleration.

Rolling motion

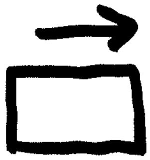
Rotational Kinetic Energy

First: Translational kinetic Energy

Translational	Rotational
x	θ
v	ω
a	α
m	I

Translational $K.E = \frac{1}{2}mv^2$

Rotational $K.E = \frac{1}{2}I\omega^2$



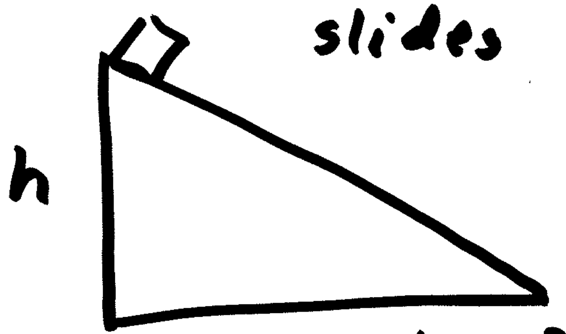
$\frac{1}{2}mv^2$

$\frac{1}{2}I\omega^2$

$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

slides

conservation of Energy



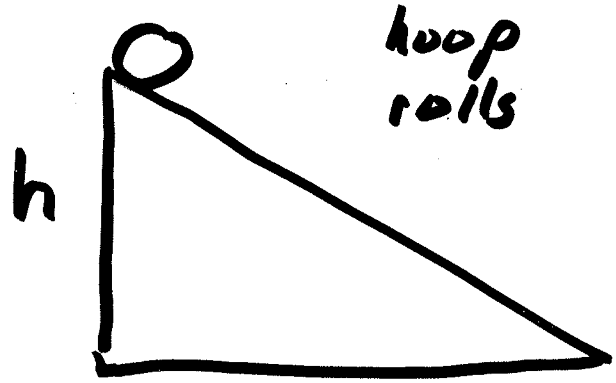
$$mgh = \frac{1}{2}mv^2$$

(frictionless)

$$\boxed{v = \sqrt{2gh}}$$

ind of mass

hoop rolls



$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad I = mr^2 \quad \omega = \frac{v}{r}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\left(\frac{v^2}{r^2}\right) \qquad gh = v^2 \qquad \boxed{v = \sqrt{gh}}$$

ind mass radius

Note: velocity at bottom smaller for rolling object

some energy goes into rotation so less energy for translation

less energy → smaller velocity

Interactive Question

A hollow cylinder of mass M and radius R rolls down an inclined plane. A block of mass M slides down an identical inclined plane. If both objects are released at the same time

A) the block will reach the bottom first.
B) the cylinder will reach the bottom first.
C) the block will reach the bottom with greater kinetic energy
D) the cylinder will reach the bottom with greater kinetic energy
E) both the block and the cylinder will reach the bottom at the same time.