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Statistical Mechanics HW 7

Qualifier Question

This question is pretty simple, but we thought it would be fun to do something on Landau Ginsberg since we didn't do that much in class. Part a is on Wikipedia and then we went further by reading about the L-G theory. We got part of c/d from another Stat Mech class's web site.

Question:

According to Landau Ginsberg theory, the free energy of a superconductor near its critical point can be described in terms of a complex order parameter, which measures how deep into the superconducting phase the system is. The free energy can be expressed as:

$$F = F_0 + F_2\phi^2 + F_4\phi^4 + F_6\phi^6 + \ldots \qquad (1)$$

Here, the F_0 term represents free energy in the normal phase, and additional terms come from the superconductor's phenomenology/properties/external B field.

a) Use this equation to calculate the behavior of the order parameter below the critical point, $\phi(T)$, keeping up to the fourth order term.

b) What effect, if any, does keeping the fourth order term in part a) above have on β?

Similarly, assume a ferroelectric has free energy:

$$F = a(T - T_c)P^2 + bP^4 + cP^6 + DxP^2 + Ex^2 \qquad (2)$$

Here, P represents electric polarization and x represents strain.

c) Find the expression that will minimize the strain.

d) When will a first order phase transition occur in this system (when does the model break down)?

Solution:

a) To determine the behavior of the variable (realizing that ϕ represents the order parameter), we minimize the order parameter by taking the derivative, setting it to zero, and solving for the parameter. This part is pretty simple:

$$\begin{aligned} F &= F_0 + F_2\phi^2 + F_4\phi^4 + F_6\phi^6 + \ldots \\ \frac{dF}{d\phi} &= 2F_2\phi + 4F_4\phi^3 \\ 0 &= 2F_2\phi + 4F_4\phi^3 \\ -F_2\phi &= 4F_4\phi^3 \end{aligned} \tag{1}$$

We see that the equation will yield two solutions:

$$\begin{aligned} \phi &= 0 \\ \phi &= \pm\sqrt{\frac{-F_2}{2F_4}} = \pm\left(\frac{-F_2}{2F_4}\right)^{1/2} \end{aligned} \tag{2}$$

b) The other part of physics in this first part of the question is seeing that for free energy in the Landau-Ginsberg model, $\phi \sim x^{\beta}$

Knowing this, we can see that the model predicts that $\beta = \frac{1}{2}$.

To see if keeping the fourth order parameter affected this, we recalculate, considering the fourth parameter and up to be zero:

$$\begin{aligned} F &= F_0 + F_2\phi^2 + F_4\phi^4 + F_6\phi^6 + \ldots \\ \frac{dF}{d\phi} &= 2F_2\phi \\ 0 &= 2F_2\phi \end{aligned} \tag{3}$$

We see that without keeping the fourth order parameter, we have only the trivial solution of $\phi = 0$, assuming the second order parameter cannot be zero. Therefore we cannot find β without keeping the fourth order term.

c) Minimizing the strain, we know now, will follow the same general process that minimizing the order parameter took previously. First, we simply solve for the extrema of the strain by setting the derivative with respect to x equal to zero:

$$F = a(T - T_c)P^2 + bP^4 + cP^6 + DxP^2 + Ex^2$$
$$\frac{dF}{dx} = DP^2 + 2Ex$$
$$0 = DP^2 + 2Ex \tag{4}$$
$$2Ex = -DP^2$$
$$x = \frac{-DP^2}{2E}$$

Now to find the expression to minimize strain, we substitute this value for x back in to the original equation for the free energy:

$$F = a(T - T_c)P^2 + bP^4 + cP^6 + D\left(\frac{-DP^2}{2E}\right)P^2 + E\left(\frac{-DP^2}{2E}\right)^2$$
$$F = a(T - T_c)P^2 + bP^4 + cP^6 - \frac{D^2}{2E}P^4 + E\frac{D^2P^4}{4E^2}$$
$$F = a(T - T_c)P^2 + \left(b - \frac{D^2}{2E} + \frac{D^2}{4E}\right)P^4 + cP^6 \tag{5}$$
$$F = a(T - T_c)P^2 + \left(b - \frac{D^2}{4E}\right)P^4 + cP^6$$

From the physics, we know that a phase transition can occur where the model breaks down. From the first part of the question, we learned that we need the fourth order parameter in order to accurately represent the free energy. And from the calculations, we saw that the fourth order term will go to zero and then become negative (also bad) when:

$$b \le \frac{D^2}{4E}$$