Problem 4 (10 Points):

Assume that air obeys the ideal gas equation. Take M to be the molar mass, P the pressure, R the ideal gas constant, T the temperature, z the altitude, ρ the density, and g the acceleration due to gravity.

a. The density of our atmosphere decreases with increasing altitude. This is a consequence of hydrostatic equilibrium, where the pressure of the air at an altitude z, must balance the pressure from below and the weight of the column of air above. Given that air has a mass density $\rho = MP/RT$, find dP/dz. Assume that the atmosphere is isothermal. Neglect the curvature of the earth and the variation of g with altitude. **(4 Points)**

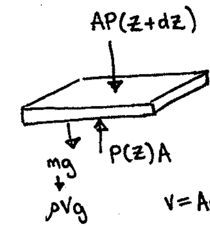
b. Using the model in part (a.), consider a volume of air that is moved adiabatically within the atmosphere and able to do work on its surroundings; that is, expand and contract to maintain the same pressure as the surrounding air. If this section is moved upwards, it will cool as it is lifted, thus increasing in density compared to the surrounding air, and tend to sink back to its original altitude. Find dT/dz, the adiabatic lapse rate for the air. Assume the air is composed of diatomic molecules (N_2). (Hint: first find dT/dP). **(4 Points)**

The significance of the adiabatic lapse rate is that it determines the stability of the atmosphere to convection. The temperature in the lower part of the real atmosphere (troposphere) is not isothermal, but decreases with increasing altitude because it is heated by the ground. If the temperature gradient in the atmosphere is greater than the lapse rate, convection can occur.

c. If the section of air was wet so that condensation can occur, how does the lapse rate change? Explain your reasoning. **(1 Points)**

d. A helium balloon ascends in the atmosphere, expanding adiabatically just as the section of air in (b.). Will the lapse rate of helium be higher, the same, or lower than air? Explain. **(1 Points)**

a)



$V = Adz$

$\Sigma F = 0 = -AP(z+dz) + AP(z) - \rho A g dz$ cancel A

Taylor expand $P(z+dz)$:

$0 = -P(z) - dz\frac{dP}{dz} + P(z) - \rho g dz$

$\frac{dP}{dz} = -\rho g = -\frac{MP}{RT} g$

b) Can't just integrate from a), because T depends on z as well.

Air is diatomic:

$E = \frac{5}{2}NkT \quad dE = \frac{5}{2}NkdT = -PdV$

$PV = NkT$

$PdV + VdP = NkdT$ $\quad V = \frac{NkT}{P}$

Combine:

$NkdT - VdP = -\frac{5}{2}NkdT$

$VdP = \frac{7}{2}NkdT$

$\frac{dT}{dP} = \frac{2V}{7Nk} = \frac{2T}{7P}$ $\quad \rho = \frac{MP}{RT}$

$\frac{dT}{dz} = \frac{\partial T}{\partial P}\frac{\partial P}{\partial z} = \left(\frac{2V}{7Nk}\right)(-\rho g)$

$= -\frac{2MPVg}{7NkRT}$ $\quad PV = NkT$

$= -\frac{2Mg}{7R}$ depends only on molar mass! makes sense ✓

c) Condensation would effectively increase molar mass, so T would fall off quicker with elevation gain in that case

d) $M_{He} < M_N$, so T of balloon would decrease more slowly with elevation gain.